# Interactive computer graphics

# Example exam questions

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1. Given two vectors $\vec{v\_{1}}=(1,2,3)$ and $\vec{v\_{2}}=(3,2,3)$ calculate the following:
	1. The dot (scalar) product of vectors: $\vec{v\_{1}}\*\vec{v\_{2}}$
	2. The cross (vector) product of vectors: $\vec{v\_{1}}×\vec{v\_{2}}$
	3. The length of each vector
	4. Normalise both vectors
	5. The angle of both vectors
	6. The projection of vector $v\_{1}$ on vector $v\_{2}$
2. Given vectors $\vec{v\_{1}}=(3,3)$ and $\vec{v\_{2}}=\left(2,3\right)$, determine the reflected vector $\vec{r}$ which is calculated as $v\_{1}$ reflected over $\vec{v\_{2}}$.
3. Given two matrices $M\_{1}=\left[\begin{matrix}1&2&3\\3&4&5\\6&6&7\end{matrix}\right]$ and $M\_{1}=\left[\begin{matrix}2&7&6\\1&2&6\\1&5&2\end{matrix}\right]$ calculate their product.
4. If a triangle is defined with vertices $A=(1,1,0)$, $B=(6,11,2)$ and $C=(11,1,0)$. Calculate the barycentric coordinates of point $T=(6,6,1)$. If point *S* has barycentric coordinates equal to $(0.5,0.5,0)$, determine the real coordinates of that point.
5. Given points $A=\left(-2,4\right)$ and $B=(1,2)$ calculate the analytic equation of the line which is defined by those two points. Additionally, calculate also the parametric line equation of the line defined by those two points. Given point $T=\left(1,1\right)$ determine whether it is located above or under the line. In addition, calculate the distance from point *T* to the line.
6. If the 2D screen is defined to have a resolution of 10 by 10 pixels, and the starting point of a line segment is $P\_{1}=(1,1)$ while the end of the line segment is $P\_{1}=\left(4,6\right)$.
	1. Sketch which pixels would be coloured when drawing a line between those two points (starting from $P\_{1}$ and going to $P\_{2}$) by using the Bresenham algorithm (the version which supports the drawing all angles).
	2. Sketch which pixels would be coloured when drawing a line between those two points (starting from $P\_{1}$ and going to $P\_{2}$) by using the Bresenham algorithm which supports drawing lines only between 0-45°. Would this algorithm draw anything? If yes, under which angle would it draw the line?
	3. Sketch which pixels would be coloured when drawing a line between those two points (starting from $P\_{1}$ and going to $P\_{2}$) by using the Bresenham algorithm (the version which supports the drawing all angles).
7. A polygon is defined by the following points: $P\_{1}=\left(50, 25\right)$, $P\_{2}=\left(25, 50\right)$, $P\_{3}=\left(25, 100\right)$, $P\_{4}=\left(50, 100\right)$, $P\_{5}=\left(100, 50\right)$.
	1. Calculate the equations of all edges of this polygon.
	2. By using the cross product between the edges and vertices of the polygon, determine whether it is convex
	3. By using the algorithm from the lab assignment determine whether points $A=(50, 50)$, $B=(25,25)$ and $C=(90,80)$ are inside or outside of a polygon.
	4. How would it be possible to determine whether a point lies on an edge of the polygon when the algorithm in the previous sub question is used.
	5. Sketch the polygon.
	6. Fill the polygon by using the algorithm implemented in the laboratory assignment. Outline which edges are „left“ and which are „right“.
8. A polygon is defined by the following points: $P\_{1}=\left(0,0,0\right)$, $P\_{2}=\left(100,0,0\right)$, $P\_{3}=\left(50, 0,100\right)$. Calculate the intersection between the polygon and the line defined with points $T\_{1}=\left(50,25,10\right)$, $T\_{2}=\left(50,25,-10\right)$.
9. Three points in 3D space are defined $P\_{1}=(2,1,1)$, $P\_{2}=(3,2,4)$ and $P\_{3}=(5,-1,-2)$.
	1. Calculate the analytic equation of the plane defined by those three points.
	2. Calculate the parametric equation defined by those three points.
	3. Determine the normal vector of the plane defined by those three points.
	4. Determine whether the point is located above or under the given plane.
	5. Determine the distance between the plane defined by those three points and point $T=(5,5,5)$.
10. A definition of a 3D convex body is given in figure 1. The definition is defined as in regular .obj file from the lab assignment.

# unknown\_model.obj

v 1.00 1.00 1.00

v 2.00 1.00 1.00

v 1.00 2.00 1.00

v 1.00 1.00 2.00

f 1 3 2

f 1 4 3

f 1 2 4

f 2 3 4

* 1. Calculate the equations of all planes which form the body.
	2. By using the algorithm described in the lab assignment, determine whether point $P=\left(0.5,1,2\right)$ is inside or outside the body.
	3. By using the algorithm described in the lab assignment, how would one determine whether the point lies exactly on the surface of the body.

Figure 1 Body definition

1. Determine the transformation matrix which will, when applied to the image on the left will result in the image denoted on the right. The top and bottom coordinates of the image before and after the transformation are denoted in the images.

 

Solution:

 As it can be seen from the example, after the transformation the position of the image and its rotation are different. This means that we need to perform the following two transformations: translation and rotation.

The translation matrix is defined as

$$T=\left[\begin{matrix}1&0&0\\0&1&0\\T\_{x}&T\_{y}&1\end{matrix}\right]$$

Where $T\_{x}$ and $T\_{y}$ represent the amount by which the points should be translated.

The rotation matrix is defined as

$$R=\left[\begin{matrix}\cos((α))⁡&\sin((α))&0\\-\sin((α))⁡&\cos((α))&0\\0&0&1\end{matrix}\right]$$

Which rotates the points counter clockwise by the given angle **around the origin**.

By using these two transformation matrices, we can construct the matrix which when applied to the first image will result in the second image. So we just need to determine the order in which we will apply these transformations. Since we need to rotate the image, and the image is rotated around the origin of the system, the first thing we need to do is to translate the centre of the image into the origin of our coordinate system. Based on the given points, we see that the center of the left image is in (8,-5). Since we need to move it to (0,0) we need to translate the first coordinate by -8, and the second one by 5. Thus our first translation matrix would be equal to:

$$T\_{1}=\left[\begin{matrix}1&0&0\\0&1&0\\-8&5&1\end{matrix}\right]$$

Now that the object is in the origin, we need to rotate it. We see that the image is rotated by 90°in clockwise direction. However, since our transformation matrix performs the transformation in the counter clockwise direction, we need to rotate it by 270° in that direction. Thus our rotation matrix would be equal to:

$$R=\left[\begin{matrix}\cos((270))⁡&\sin((270))&0\\-\sin((270))⁡&\cos((270))&0\\0&0&1\end{matrix}\right]=\left[\begin{matrix}0&1&0\\1&0&0\\0&0&1\end{matrix}\right]$$

Finally, we need to translate the image to the right location. We see that the centre of the image on the right side is located at (5,3). Thus, we need to translate the centre of our rotated image (which is currently at the origin) by 5 for the first coordinate, and by 3 for the second coordinate. Thus our second translation matrix will be equal to:

$$T\_{2}=\left[\begin{matrix}1&0&0\\0&1&0\\5&3&1\end{matrix}\right].$$

Finally, by multiplying all three matrices we obtain the matrix which represents the complete transformation.

$$T=\left[\begin{matrix}1&0&0\\0&1&0\\-8&5&1\end{matrix}\right]\*\left[\begin{matrix}0&-1&0\\1&0&0\\0&0&1\end{matrix}\right]\*\left[\begin{matrix}1&0&0\\0&1&0\\5&3&1\end{matrix}\right]=\left[\begin{matrix}0&-1&0\\1&0&0\\10&11&1\end{matrix}\right]$$

We can no check that by multiplying the centre of the left image we will really obtain the center of the right image.

$$P\_{r}=P\*T=(5,3,1)$$

As an exercise try to determine the matrix which would perform the inverse transformation and see how it is correlated with the matrix which we calculated in this example.