

# OTKRIVANJE FIZIKALNIH IZRAZA GENETSKIM PROGRAMIRANJEM

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# UVOD

- Pronalaženje jednačbi koje opisuju podatke
- Jednačbe u obliku  $y = f(\mathbf{X})$
- Predstavljaju jednačbe iz fizike

# SIMBOLIČKA REGRESIJA

- Problem pronalaženja simboličkog izraza koji najbolje opisuje podatke
- Teži od obične regresije
- Traži se model a ne parametri modela

# GENETSKO PROGRAMIRANJE

- Proširenje genetskog algoritma
- Kao jedinke se koriste prikladnije strukture poput stabala
- Elitizam
- Napuhivanje (*eng. bloat*)
- Nije optimalan algoritam

# IMPLEMENTACIJA

- Implementacija u programskom jeziku java
- Implementirane su generacijska i 3-turnirska eliminacijska inačica algoritma
- Jedinke su stabla

# OPERATORI KRIŽANJA

- ◉ Zamjena podstabala
- ◉ Ujedinjenje stabala

# OPERATORI MUTACIJE

- ◉ Mutacija izrastom
- ◉ Mutacija zamjenom čvora terminalnim
- ◉ Mutacija uzimanjem podstabla
- ◉ Mutacija zamjenom tipa čvora
- ◉ Mutacija potpunom zamjenom

# FUNKCIJE DOBROTE

- Srednje kvadratno odstupanje
- Normalizirano srednje kvadratno odstupanje



# KORIŠTENI PARAMETRI

- Veličina populacije od 50 jedinki
- Vjerojatnost mutacije 0.9
- Vjerojatnost križanja u generacijskom algoritmu 0.2
- Provedeno je 1000 iteracija algoritma

# REZULTATI

- Pronalaskom se smatra stablo koje definira izraz ekvivalentan traženoj jednažbi, do na pomak i skaliranje
- Od 100 jednažbi
  - 30 ih pronađe algoritam sa srednjim kvadratnim odstupanjem
  - 53 ih pronađe algoritam sa normaliziranim srednjim kvadratnim odstupanjem
- Generacijski algoritam je uspješniji od eliminacijskog
  - 11/30 i 41/52

# PRIMJERI REZULTATA

⊙ Tražena jednačba:  $y = x_0 x_1$

- $((x_1)*(x_0))$
- $((x_0)*(x_1))-((x_0)-(x_0))$
- $((-((-((-x_1)))-(x_1))))*(x_0)$

⊙ Tražena jednačba:  $y = \frac{1}{\sqrt{2\pi}} e^{-\frac{x_0^2}{2}}$

- $((4.115724768351486)*(sqrt((exp((-((x_0)*(x_0))))))))$
- $(exp(((3.1262938707654797)+(((x_0)/(-2.0))*(x_0))))$
- $((((Sin((x_0)))*((x_0)+(sqrt(((sqrt(((1.4142135623730951)*(x_0)))))*(x_0))+((x_0)/((x_0)/((x_0)*(-0.3914338389102179))))))))-(((x_0)-(1.0))/(exp((6.283185307179586))))+((x_0)/((0.10395099821277753)+(x_0)))))/(((((((x_0)/(x_0))*(x_0))/((((((x_0)+(x_0))-(x_0)))^(-1))/(x_0))*(x_0)-(((((((x_0))^2)*(ln((x_0))))/((exp((Sin((x_0)))))/(-4.887100276723293))-Sin((x_0))))))^2))$

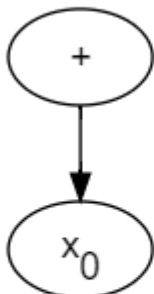
HVALA NA PAŽNJI

Ime čvora	Broj djece	Matematička operacija
Konstanta	0	$c$
Varijabla	0	$x$
Negacija	1	$-x$
Sinus	1	$\sin(x)$
Kosinus	1	$\cos(x)$
Inverzija	1	$x^{-1}$
Kvadriranje	1	$x^2$
Korijen	1	$\sqrt{x}$
Eksponenciranje	1	$e^x$
Logaritam	1	$\ln(x)$
Zbrajanje	2	$x + y$
Oduzimanje	2	$x - y$
Množenje	2	$x * y$
Dijeljenje	2	$\frac{x}{y}$

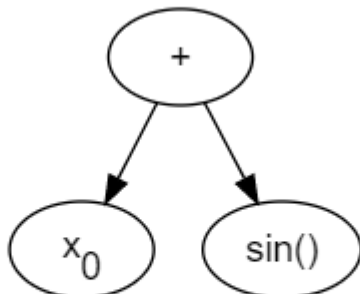
Prvi korak



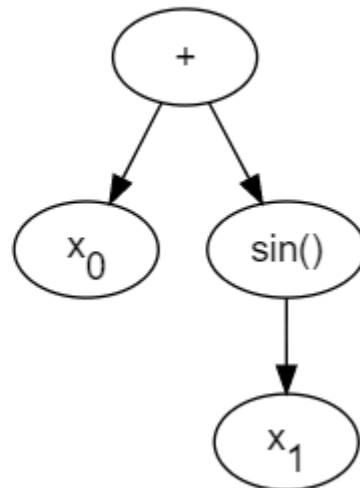
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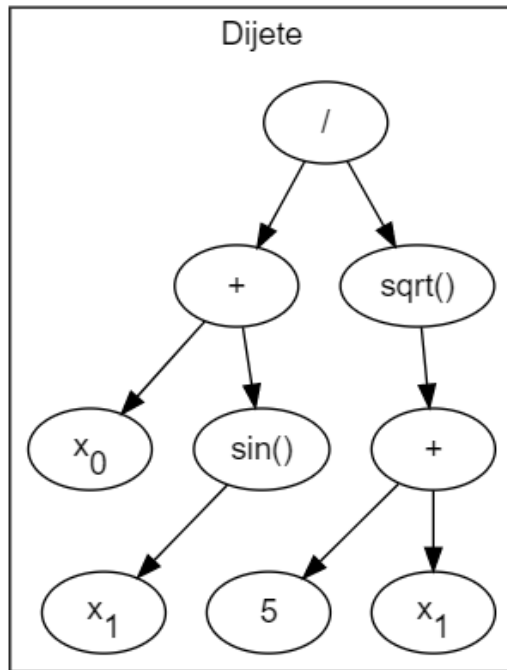
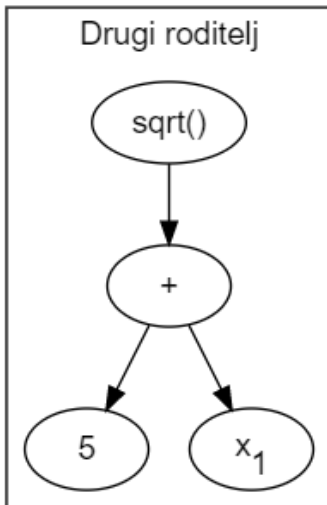
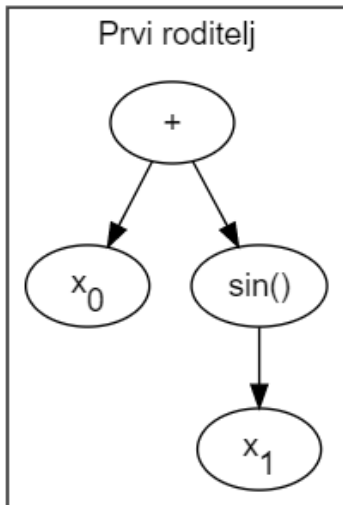
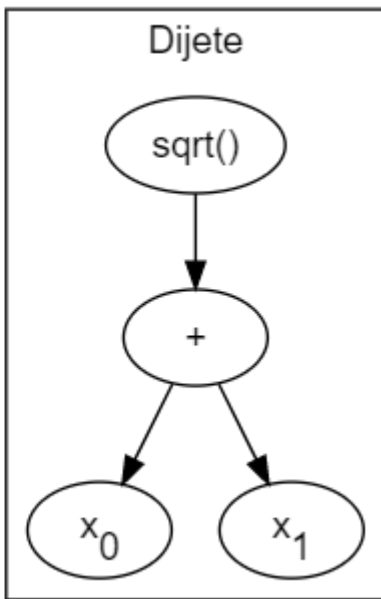
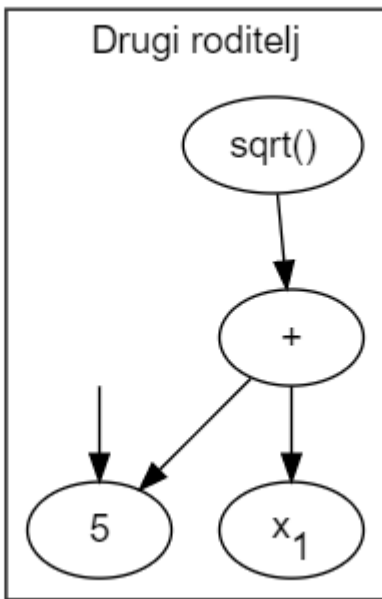
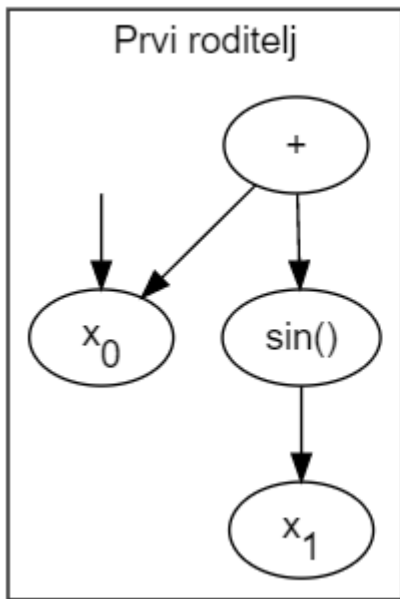


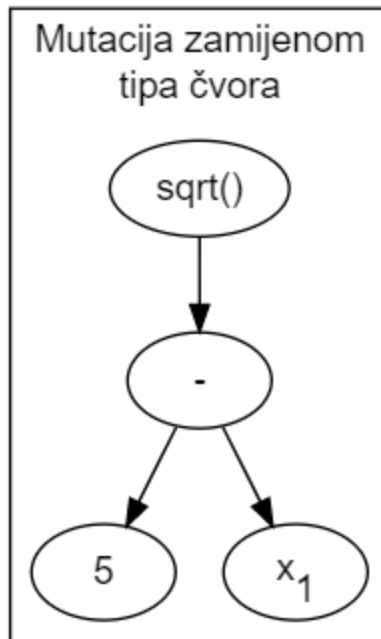
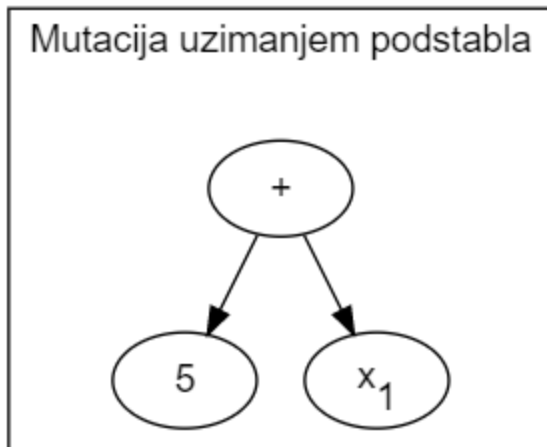
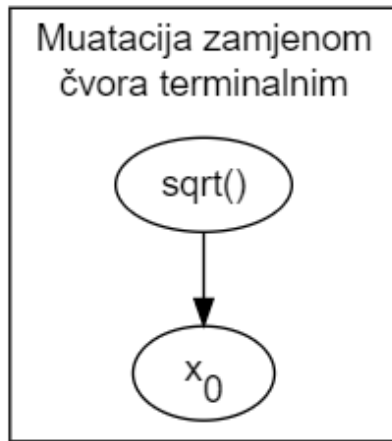
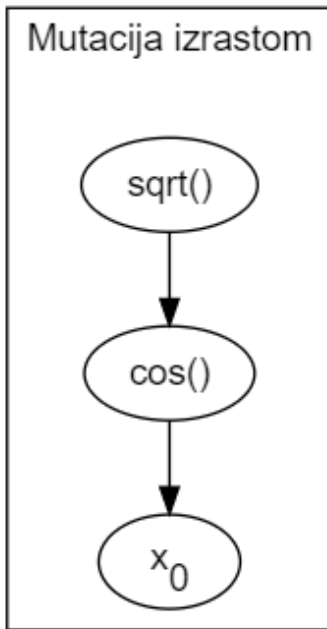
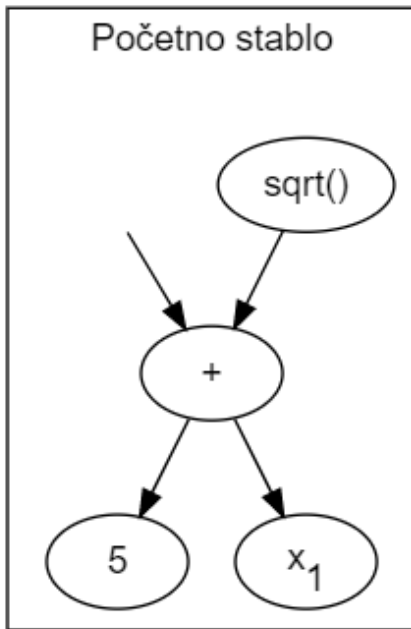
Treći korak



Četvrti korak









Jednadžba	mse	normalizirani mse
$\frac{1}{\sqrt{2\pi}} e^{-\frac{x_0^2}{2}}$	Ne/Ne	Ne/Da
$\frac{1}{\sqrt{2\pi x_1}} e^{-\frac{x_0^2}{2x_1^2}}$	Ne/Ne	Ne/Ne
$\frac{1}{\sqrt{2\pi x_1}} e^{-\frac{(x_0-x_2)^2}{2x_1^2}}$	Ne/Ne	Ne/Ne
$\sqrt{(x_1 - x_0)^2 + (x_3 - x_2)^2}$	Ne/Ne	Ne/Ne

$\frac{x_0 x_2 + x_1 x_3}{x_0 + x_1}$	Ne/Ne	Ne/Ne
$x_0 x_1 \sin(x_2)$	Ne/Da	Da/Da
$x_0 x_1 x_2 \sin(x_3)$	Ne/Da	Da/Da
$\frac{1}{4} x_0 (x_1^2 + x_2^2) x_3^2$	Ne/Ne	Ne/Ne
$\frac{x_0}{x_1}$	Da/Da	Da/Da
$\arcsin(x_0 \sin(x_1))$	Ne/Ne	Ne/Ne
$\frac{1}{\frac{1}{x_0} + \frac{1}{x_1}}$	Ne/Da	Ne/Da
$\frac{x_0}{x_1}$	Da/Da	Da/Da
$\sqrt{x_0^2 + x_1^2 - 2x_0 x_1 \cos(x_2 - x_3)}$	Ne/Ne	Ne/Ne
$x_0 \frac{\sin(\frac{x_2 x_1}{2})^2}{\sin(\frac{x_1}{2})^2}$	Ne/Ne	Ne/Ne
$\arcsin(\frac{x_0}{x_1 x_2})$	Ne/Ne	Ne/Ne
$\frac{x_0^2 x_1^2}{6\pi x_2 x_3^3}$	Ne/Ne	Da/Ne
$(\frac{1}{2} x_0 x_1 x_2^2) \frac{8\pi x_3^2}{3} \frac{x_4^4}{x_4^2 - x_3^2}$	Ne/Ne	Ne/Ne

$\frac{x_0 x_1 x_2}{\sqrt{(x_4 - x_3)^2 + (x_6 - x_5)^2 + (x_8 - x_7)^2}}$	Ne/Ne	Ne/Ne
$\frac{x_0}{\sqrt{1 - \frac{x_1^2}{x_2^2}}}$	Ne/Ne	Ne/Ne
$x_0 x_3 + x_1 x_4 + x_2 x_5$	Ne/Ne	Ne/Ne
$x_0 x_1$	Da/Da	Da/Da
$\frac{x_0 x_1}{4\pi x_2 x_3^2}$	Ne/Ne	Da/Da
$\frac{x_0}{4\pi x_1 x_2^2}$	Ne/Ne	Da/Da
$x_0 x_1$	Da/Da	Da/Da
$x_0 (x_1 + x_2 x_3 \sin(x_4))$	Ne/Ne	Ne/Ne
$\frac{1}{2} x_0 (x_1^2 + x_2^2 + x_3^2)$	Ne/Ne	Ne/Ne
$x_4 x_0 x_1 (\frac{1}{x_3} - \frac{1}{x_2})$	Ne/Ne	Ne/Ne
$x_0 x_1 x_2$	Da/Da	Da/Da
$\frac{1}{2} x_0 x_1^2$	Ne/Da	Da/Da
$\frac{x_0 - x_1 x_3}{\sqrt{1 - \frac{x_1^2}{x_2^2}}}$	Ne/Ne	Ne/Ne
$\frac{x_3 - \frac{x_0 x_3}{x_2^2}}{\sqrt{1 - \frac{x_1^2}{x_2^2}}}$	Ne/Ne	Ne/Ne
$\frac{x_0 x_1}{\sqrt{1 - \frac{x_1^2}{x_2^2}}}$	Ne/Ne	Ne/Ne
$\frac{x_1 + x_2}{\sqrt{1 + \frac{x_1 x_2}{x_0^2}}}$	Ne/Ne	Ne/Ne

$\frac{x_0 x_1 x_2}{x_3}$	Ne/Da	Da/Da
$\frac{x_2}{1 - \frac{x_1}{x_0}}$	Ne/Ne	Ne/Da
$\frac{1 + \frac{x_1}{x_0}}{\sqrt{1 - \frac{x_1^2}{x_0^2}}} x_2$	Ne/Ne	Ne/Ne
$\frac{x_1}{2\pi} x_0$	Ne/Da	Da/Da
$x_0 + x_1 + 2\sqrt{x_0 x_1} \cos(x_2)$	Ne/Ne	Ne/Ne
$\frac{4\pi x_2 x_3^2}{x_0 x_1^2}$	Ne/Ne	Da/Da
$\frac{3}{2} x_0 x_1$	Ne/Da	Da/Da
$\frac{1}{1-x_0} x_1 x_2$	Ne/Ne	Ne/Da
$\frac{x_0 x_1 x_3}{x_2}$	Ne/Da	Da/Da
$x_0 e^{-\frac{x_1 x_2 x_4}{x_3 x_5}}$	Ne/Ne	Ne/Ne
$\frac{x_2 x_0^3}{\pi^2 x_4^2 (e^{x_3 x_1} - 1)}$	Ne/Ne	Ne/Ne
$\frac{x_0 x_1 x_2}{x_3}$	Da/Da	Da/Da
$x_0 x_1 x_2$	Da/Da	Da/Da
$\frac{1}{x_0 - 1} \frac{x_1 x_3}{x_2}$	Ne/Ne	Ne/Ne
$x_0 x_1 x_2 \ln\left(\frac{x_4}{x_3}\right)$	Ne/Ne	Ne/Ne
$\sqrt{\frac{x_0 x_1}{x_2}}$	Ne/Ne	Da/Da
$\frac{x_0 x_2^2}{\sqrt{1 - \frac{x_1^2}{x_2^2}}}$	Ne/Ne	Ne/Ne

$x_0 [\cos(x_1 x_2) + x_3 \cos(x_1 x_2)^2]$	Ne/Ne	Ne/Ne
$\frac{x_0 (x_2 - x_1) x_3}{x_4}$	Ne/Ne	Ne/Da
$\frac{x_0}{4\pi x_5^2}$	Ne/Ne	Da/Da
$\frac{x_0}{4\pi x_1 x_2}$	Ne/Ne	Da/Da
$\frac{1}{4\pi x_0} \frac{x_1 \cos(x_2)}{x_3^2}$	Ne/Ne	Ne/Da
$\frac{3}{4\pi x_0} \frac{x_1 x_5}{x_2^2} \sqrt{x_3^2 + x_4^2}$	Ne/Ne	Ne/Ne
$\frac{3}{4\pi x_0} \frac{x_1}{x_3} \cos(x_2) \sin(x_2)$	Ne/Ne	Ne/Da
$\frac{3}{5} \frac{x_0^2}{4\pi x_1 x_2}$	Ne/Ne	Da/Da
$\frac{x_0 x_1^2}{2}$	Ne/Da	Da/Da
$\frac{x_0}{x_1} \frac{1}{1+x_2}$	Ne/Da	Ne/Da
$\frac{x_0 x_1}{x_2 (x_3^2 - x_4^2)}$	Ne/Ne	Ne/Ne
$x_0 \left(1 + \frac{x_4 x_5 \cos(x_3)}{x_1 x_2}\right)$	Ne/Ne	Ne/Ne
$\frac{x_0 x_1^2 x_2}{3 x_3 x_4}$	Ne/Ne	Da/Da
$\frac{x_0 x_1}{1 - \frac{x_0 x_1}{3}} x_2 x_3$	Ne/Ne	Ne/Ne

$1 + \frac{x_0 x_1}{1 - \frac{x_0 x_1}{3}}$	Ne/Ne	Ne/Ne
$\frac{1}{4\pi x_0 x_1^2} \frac{2x_2}{x_3}$	Ne/Da	Ne/Da
$\frac{x_0}{\sqrt{1 - \frac{x_1^2}{x_2^2}}}$	Ne/Ne	Ne/Ne
$\frac{x_0 x_1}{\sqrt{1 - \frac{x_1^2}{x_2^2}}}$	Ne/Ne	Ne/Ne
$-x_0 x_1 \cos(x_2)$	Ne/Da	Da/Da
$-x_0 x_1 \cos(x_2)$	Ne/Da	Ne/Da
$\frac{x_0}{4\pi x_1 x_2 (1 - \frac{x_3}{x_4})}$	Ne/Ne	Ne/Ne
$\sqrt{\frac{x_0^2}{x_1^2} - \frac{\pi^2}{x_2^2}}$	Ne/Ne	Ne/Ne
$x_0 x_1 x_2^2$	Ne/Da	Da/Da
$x_0 x_1^2$	Da/Da	Da/Da
$\frac{x_0 x_1}{2\pi x_2}$	Ne/Ne	Da/Da
$\frac{x_0 x_1 x_2}{2}$	Ne/Da	Da/Da
$\frac{x_0 x_1 x_2}{2x_3}$	Ne/Da	Da/Da
$\frac{x_0 x_1}{4\pi x_2}$	Ne/Ne	Da/Da
$\frac{x_0 x_2 x_3 x_4}{\frac{x_1}{2\pi}}$	Ne/Ne	Da/Da
$\frac{x_0}{e^{\frac{x_3 x_4}{x_1 x_2}} + e^{-\frac{x_3 x_4}{x_1 x_2}}}$	Ne/Ne	Ne/Ne
$x_0 x_1 \tanh(\frac{x_1 x_2}{x_3 x_4})$	Ne/Ne	Ne/Ne
$\frac{x_0 x_1}{x_2 x_3} + \frac{x_0 x_4 x_7}{x_5 x_6^2 x_2 x_3}$	Ne/Ne	Ne/Ne
$x_0(1 + x_2)x_1$	Da/Da	Da/Da
$\frac{x_0 x_1 x_3}{x_2}$	Ne/Da	Da/Da
$\frac{x_0}{2(1+x_1)}$	Ne/Da	Ne/Da

$\frac{1}{e^{\frac{x_0}{2\pi} \frac{x_1}{x_2 x_3}} - 1}$	Ne/Ne	Ne/Ne
$\frac{\frac{x_0}{2\pi} x_1}{e^{\frac{x_0}{2\pi} \frac{x_1}{x_2 x_3}} - 1}$	Ne/Ne	Ne/Ne
$\frac{2x_0 x_1}{\frac{x_2}{2\pi}}$	Ne/Ne	Da/Da
$\sin(\frac{x_0 x_1}{\frac{x_2}{2\pi}})^2$	Ne/Ne	Ne/Ne
$\frac{\frac{x_0 x_1 x_2}{\frac{x_3}{2\pi}} \sin((x_4 - x_5) \frac{x_2}{2})^2}{((x_4 - x_5) \frac{x_2}{2})^2}$	Ne/Ne	Ne/Ne
$x_0 \sqrt{x_1^2 + x_2^2 + x_3^2}$	Ne/Ne	Ne/Ne
$x_0 \frac{x_1}{2\pi}$	Da/Da	Da/Da
$\frac{2x_0 x_1^2 x_2}{\frac{x_3}{2\pi}}$	Ne/Ne	Da/Da
$x_0(e^{\frac{x_1 x_2}{x_3 x_4}} - 1)$	Ne/Ne	Ne/Ne

$2x_0(1 - \cos(x_1 x_2))$	Ne/Ne	Ne/Da
$\frac{(\frac{x_0}{2\pi})^2}{2x_1 x_2^2}$	Ne/Ne	Da/Da
$\frac{2\pi x_0}{x_1 x_2}$	Da/Da	Da/Da
$x_0(1 + x_1 \cos(x_2))$	Ne/Da	Da/Da
$\frac{-x_0 x_1^4}{2(4\pi x_4)^2 (\frac{x_2}{x_3})^2} \frac{1}{x_3}$	Ne/Ne	Ne/Ne
$\frac{-x_0 x_1 x_2}{x_3}$	Ne/Ne	Da/Da