Error Analysis of a Stewart Platform Based Manipulators

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***Abstract* – Hexapod characteristic analyzed in this paper is the effect of small errors within its elements (strut lengths, joint placement) which can be caused by manufacturing tolerances or setting up errors or other even unknown sources to end effector. If all error values are known or can be calculated then they can be included in a model which will eliminate their defective effect. In practice there are very few error elements that are known in advance. For all others their values can just be assumed. What can be done now is to find a way to approximate value for maximal end effector displacement. The method we propose is a numerical method which is based on differential equations of close loop vector chains. The method can be used to calculate maximal end effector error for specific position and also for validation of entire hexapod workspace area, as shown in this paper.**

I. INTRODUCTION

Parallel kinematic manipulators (PKM) have recently been rediscovered as today's microprocessor's power satisfies computing force required for their control. Its great payload capacity, stiffness and accuracy characteristic as result of their parallel structure make them superior to serial manipulators in many fields. But serial manipulators are still foremost used because PKMs are mostly still under development, although there are already available such manipulators at the market.

One of the most accepted PKM is Stewart platform based manipulator, also known as hexapod or Gough platform. Hexapod, originally, consists of two platforms, one fixed on the floor or ceiling and one mobile, connected together via six extensible struts by spherical or other types of joints. That construction gives mobile platform 6-DOF (degree of freedom). Hexapod movement and control is achieved only through strut lengths changes. One variation to this structure, also observed here, is when struts are fixed in length but one of their ends is placed on guideways. Control is then obtained only by moving those joints on guideways. Although in this model the forces acting on struts aren't just along the axis of the struts, like with original design, practically attainable sliding characteristics of guideways make it very considerable structure for manipulators.

Error models and algorithms that compensate errors for conventional machine tools cannot be used with hexapod parallel structure therefore a new approaches are needed. The effect of manufacturing tolerances on the accuracy was investigated by Wang and Masory [1] by modeling the legs as serial kinematic chains. S.M. Wang and K.F. Ehmann [2] present first and second order error models for a 6-DOF Stewart Platform manipulator using differential leg length changes. A.J. Patel and K.F. Ehmann [3] present an error analysis based on error model formed through differentiation of the kinematic equations.

In this paper the last error model [3] is used, extended to other hexapod structure. Using worst case scenario an algorithm for error analysis is presented and shown on few models.

II. HEXAPOD MODELS

Standard Stewart Platform based manipulator as shown in Fig.1 can be defined in many ways but most common set of parameters are: minimal and maximal struts length ($l_{\min}$, $l_{\max}$), radii of fixed and mobile platforms (r_1, r_2), joint placement defined with angle between closest joints for both platforms (α, β) and joint moving area (assuming cone with angle γ).

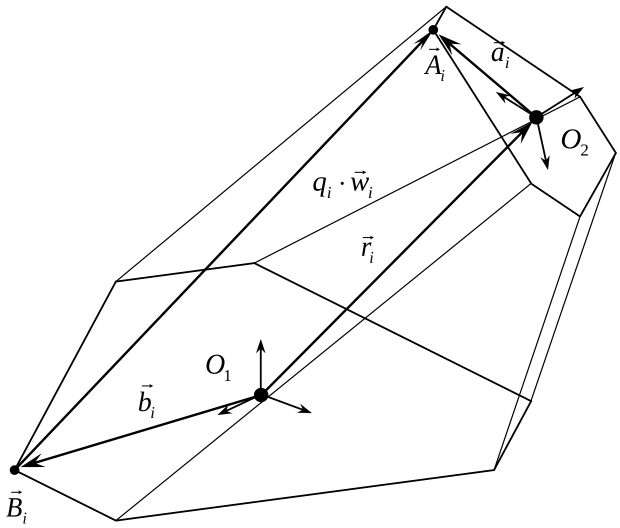

Fig. 1. Stewart Platform manipulator

Inverse kinematic can be described with equation:

$$\begin{aligned} \vec{A}_i &= \vec{T} + \underline{R} \cdot {}^m\vec{A}_i, \\ l_i &= d\left(\vec{A}_i, \vec{B}_i\right), \end{aligned} \tag{1}$$

where $\vec{A}_i$ and $\vec{B}_i$ are joint position vectors on base and mobile platform, ${}^m\vec{A}_i$ are joint position vectors of mobile platform in local coordinate system, $\vec{T}$ is translation vector between base and mobile systems, $\underline{R}$ is orientation matrix of mobile platform, l_i are strut lengths calculated with inverse kinematic and $d()$ is distance between two joints, at the beginning and end of struts.

The second observed hexapod model, shown in Fig.2, differs from standard Stewart manipulator at base platform and struts. Strut lengths are constant and same for all struts but their joints on one side are placed on sliding guideways where actuators are placed. Parameters which describe this model differ only for base platform where guideways are placed: $\vec{B}_{k,i}$ and $\vec{B}_{p,i}$ define i[th] guide way and t_i as value between [0, 1] identify actual joint position. If we observe models like on Fig.2, those vectors can be defined using four parameters: d as distance between closes parallel guide ways, r_{11} and r_{12} as radii of circles where guide ways ends are placed with height difference h.

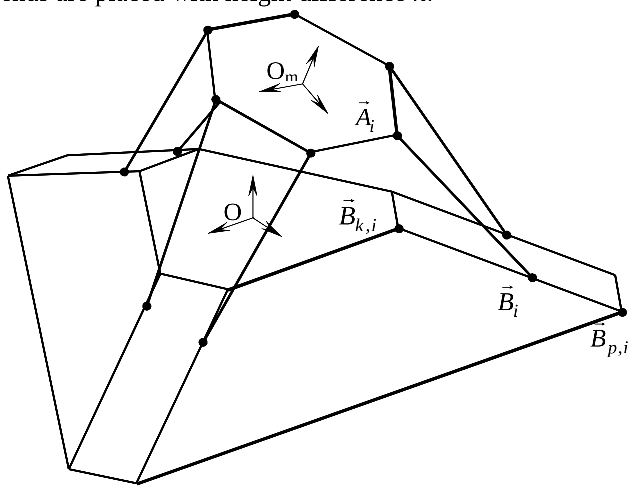

Fig. 2. Hexapod with fixed strut lengths

Inverse kinematic for this model is slightly more complex from standard hexapod and can be computed using equations:

$$\begin{aligned} \vec{A}_i &= \vec{T} + \underline{R} \cdot {}^m\vec{A}_i, \\ l &= d\left(\vec{A}_i, \vec{B}_i\right), \\ \vec{B}_i &= \vec{B}_{p,i} + s_i \cdot \left(\vec{B}_{k,i} - \vec{B}_{p,i}\right) \end{aligned} \tag{2}$$

s_i is calculated from quadratic equation and therefore can give two possible joint position on same guide way. This problem must be solved in control procedures.

End effector (tool) is placed on mobile platform above the geometrical center of joints placed on that platform by height l_{tool}. Therefore, origin of local coordinate system of mobile platform is placed in that point. Subsequently vectors ${}^m\vec{A}_i$ are calculated for that origin.

Using inverse kinematic for any point and tool orientation, it is possible to compute mobile platform position and orientation. Then struts lengths for first or joint positions for second model can be calculated. If strut lengths are within given ranges, or joints can be placed on guideways for 2[nd] model, and other constraints are fulfilled, as joint angle constraint and no collision between struts, than hexapod is capable putting its end effector in that point with given orientation. In this way a working area with given orientation can be found.

Assuming that manipulator is used for machining free surface pieces, working area can be better defined as area were manipulator can work for any required orientation. Required orientations which give optimal surface characteristics usually can be defined with vectors within a cone with defined angle as in Fig. 5. Working area calculated using this definition gives superior visual and numeric description of manipulator.

When dealing with 6-DOF hexapod manipulators, which on its end effector have tool on spindle, inverse kinematic can't generally give unique result. This gives freedom to apriori choose rotation angle of moving platform as the 6[th] DOF. For simplicity, no rotation angle was used whenever such orientation was feasible.

III. THE ERROR MODEL

Control of hexapod manipulator is based on described inverse kinematic. However, that was valid only for models. In reality, because of unpredictable environment, some hexapod elements may have values different from nominal. This can be due to the assembly errors, elastic and thermical deformations, actuator errors and others error sources. Model that includes all sources of errors is hardly possible to implement, first, because of nonlinear dependent error sources, and second, because most of error elements can't even be calculated or measured. What can be done is to give an approximate value for error at end effector if error sources are given as approximate values, just quantities, not directions.

From Fig.1, for one vector chain through i[th] strut, the folowing equation can be deducted:

$$\vec{b}_i + q_i \cdot \vec{w}_i = \vec{r} + \underline{R} \cdot {}^P\vec{a}_i. \tag{3}$$

Differentiating this equation yields:

$$\delta\vec{b}_i + \delta q_i \cdot \vec{w}_i + q_i \cdot \delta\vec{w}_i = \delta\vec{r} + \delta\underline{R} \cdot {}^P\vec{a}_i + \underline{R} \cdot \delta\, {}^P\vec{a}_i, \tag{4}$$

which can be interpreted as relations between errors in joint positions $\delta\vec{b}_i, \delta\, {}^P\vec{a}_i$ and actuator errors δq_i with errors at end effector position $\delta\vec{r}$ and orientation $\delta\underline{R}$. Furthermore, two more error elements are added to (4), errors in joint centre position, both on mobile and fixed platform:

$$\begin{aligned} &\delta\vec{b}_i + \vec{c}_i + \delta q_i \cdot \vec{w}_i + q_i \cdot \delta\vec{w}_i = \\ &\quad \delta\vec{r} + \delta\underline{R} \cdot {}^P\vec{a}_i + \underline{R} \cdot \left(\delta\, {}^P\vec{a}_i + {}^P\vec{d}_i\right) \end{aligned} \tag{5}$$

Fig.3 shows one close-loop vector chain for i[th] strut for first hexapod model with included errors, whose values are intentionally enlarged.

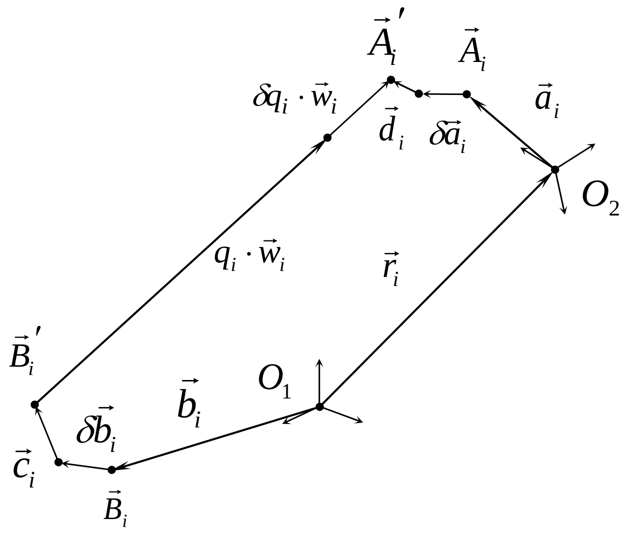

Fig. 3. A vector chain with error components (1)

Multiplying (5) with $\vec{w}_i^T$, than replacing $\delta \underline{R} = \delta\tilde{\Omega} \cdot \underline{R}$, where $\delta\Omega$ is orientation error vector, and with simple vector and mathematics transformations (5) becomes (6).

$$\begin{aligned} \delta q_i = {} & \delta\vec{r} \cdot \vec{w}_i + \delta\vec{\Omega} \cdot \left(\vec{a}_i \times \vec{w}_i\right) + \\ & + \vec{w}_i \cdot \underline{R} \cdot \left(\delta^P \vec{a}_i + {}^P\vec{d}_i\right) - \vec{w}_i \cdot \left(\delta\vec{b}_i + \vec{c}_i\right) \end{aligned} \tag{6}$$

Equation (6) can be generalized and used in matrix form:

$$\begin{aligned} \delta\vec{\Lambda} &= \underline{J} \cdot \delta\vec{\Pi} + \underline{N} \cdot \delta\vec{A}, \\ \delta\vec{\Pi} &= \underline{J}^{-1} \cdot \left(\delta\vec{\Lambda} - \underline{N} \cdot \delta\vec{A}\right), \end{aligned} \tag{7}$$

where

$$\delta\vec{\Lambda} = \left[\delta q_1 \;\delta q_2 \;\ldots \;\delta q_6\right]^T, \tag{8}$$

$$\delta\vec{\Pi} = \left[\delta\vec{r}^{\,T} \;\delta\vec{\Omega}^T\right]^T = \left[\delta r_x \;\delta r_y \;\delta r_z \;\omega_x \;\omega_y \;\omega_z\right]^T, \tag{9}$$

$$\underline{J} = \begin{bmatrix} \vec{w}_1^T & \left(\vec{a}_1 \times \vec{w}_1\right)^T \\ \vdots & \vdots \\ \vec{w}_6^T & \left(\vec{a}_6 \times \vec{w}_6\right)^T \end{bmatrix} \delta\vec{A} = \begin{bmatrix} {}^P\delta\vec{a}_1 + {}^P\vec{d}_1 \\ \delta\vec{b}_1 + \vec{c}_1 \\ \vdots \\ {}^P\delta\vec{a}_6 + {}^P\vec{d}_6 \\ \delta\vec{b}_6 + \vec{c}_6 \end{bmatrix} \in \Re^{36\times 1}, \tag{10}$$

$$\underline{N} = \begin{bmatrix} \vec{w}_1^T \cdot \underline{R} & -\vec{w}_1^T & \cdots & \vec{0}^T & \vec{0}^T \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \vec{0}^T & \vec{0}^T & \cdots & \vec{w}_6^T \cdot \underline{R} & -\vec{w}_6^T \end{bmatrix} \in \Re^{6\times 36}. \tag{11}$$

With formula (7) error in position and orientation at end effector can be calculated if all errors are known or at least presumed.

Formulas for the second hexapod model can be achieved following the same procedure. First, from Fig.2 subsequent equation can be written:

$$\vec{b}_i + s_i \cdot \vec{l}_i + q_i \cdot \vec{w}_i = \vec{r} + \underline{R} \cdot {}^P\vec{a}_i, \tag{12}$$

where $\vec{l}_i$ describe i^{th} guide way orientation. Differentiating this equation, with $\delta\vec{b}_i = \vec{0}$, $\delta\vec{l}_i = \vec{0}$ we get:

$$\delta s_i \cdot \vec{l}_i + \delta q_i \cdot \vec{w}_i + q_i \cdot \delta\vec{w}_i = \delta\vec{r} + \delta\underline{R} \cdot {}^P\vec{a}_i + \underline{R} \cdot \delta\vec{a}_i. \tag{13}$$

Adding errors in joint centre positions, both on mobile and fixed platform accordingly with Fig. 4, and replacing $\delta \underline{R} = \delta\tilde{\Omega} \cdot \underline{R}$ formula becomes:

$$\begin{aligned} & \delta s_i \cdot \vec{l}_i + \delta q_i \cdot \vec{w}_i + q_i \cdot \delta\vec{w}_i + \vec{c}_i = \\ & \qquad \delta\vec{r} + \delta\vec{\Omega} \times \vec{a}_i + \underline{R} \cdot \left({}^P\delta\vec{a}_i + {}^P\vec{d}_i\right) \end{aligned} \tag{14}$$

Equation (14) can be arranged as (15) and then written in matrix form (16) where matrices are defined with (17), (18), (19) and (20).

$$\begin{aligned} \vec{w}_i \cdot \vec{l}_i \cdot \delta s_i = {} & \vec{w}_i \cdot \delta\vec{r} + \left(\vec{a}_i \times \vec{w}_i\right) \cdot \delta\vec{\Omega} + \\ & + \vec{w}_i \cdot \underline{R} \cdot \left({}^P\delta\vec{a}_i + {}^P\vec{d}_i\right) + \vec{w}_i \cdot \vec{c}_i - \delta q_i \end{aligned} \tag{15}$$

$$\begin{aligned} \underline{S} \cdot \delta\vec{\Psi} &= \underline{J} \cdot \delta\vec{\Pi} + \underline{N} \cdot \delta\vec{A} + \vec{B} \\ \delta\vec{\Pi} &= \underline{J}^{-1} \cdot \left(\underline{S} \cdot \delta\vec{\Psi} - \underline{N} \cdot \delta\vec{A} - \vec{B}\right) \end{aligned} \tag{16}$$

Equation (16) is an equivalent for (7) for model with fixed strut lengths. But as already said exact values for each error element must be known to calculate errors at end effector.

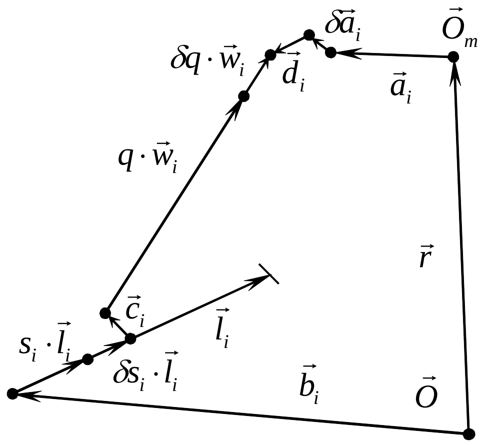

Fig. 4. A vector chain with error components (2)

$$\delta\vec{\Psi} = \begin{bmatrix} \delta s_1 \\ \vdots \\ \delta s_6 \end{bmatrix} \quad \underline{J} = \begin{bmatrix} \vec{w}_1^T & \left(\vec{a}_1 \times \vec{w}_1\right)^T \\ \vdots & \vdots \\ \vec{w}_6^T & \left(\vec{a}_6 \times \vec{w}_6\right)^T \end{bmatrix} \tag{17}$$

$$\underline{N} = \begin{bmatrix} \vec{w}_1^T \cdot \underline{R} & \cdots & \vec{0}^T \\ \vdots & \ddots & \vdots \\ \vec{0}^T & \cdots & \vec{w}_6^T \cdot \underline{R} \end{bmatrix} \in \Re^{6\times 18} \tag{18}$$

$$\underline{S} = \begin{bmatrix} \vec{w}_1^T \cdot \vec{l}_1 & \cdots & \vec{0}^T \\ \vdots & \ddots & \vdots \\ \vec{0}^T & \cdots & \vec{w}_6^T \cdot \vec{l}_6 \end{bmatrix} \tag{19}$$

$$\delta\vec{A} = \begin{bmatrix} {}^P\delta\vec{a}_1 + {}^P\vec{d}_1 \\ \vdots \\ {}^P\delta\vec{a}_6 + {}^P\vec{d}_6 \end{bmatrix} \quad \delta\vec{B} = \begin{bmatrix} \delta q_1 + \vec{w}_1 \cdot \vec{c}_1 \\ \vdots \\ \delta q_6 + \vec{w}_6 \cdot \vec{c}_6 \end{bmatrix} \tag{20}$$

What can be done if errors can only be approximated with some border values? Using worst case method and formulas (7) or (16) a maximal error can be found searching through all possible input error values. This method is presented in the next chapter.

III. THE WORST CASE METHOD

Error vector $\delta\vec{\Pi}$ for both hexapod models can be expressed in a form:

$$\delta\vec{\Pi} = \underline{J}^{-1}\left(\underline{K} \cdot \vec{x}\right), \tag{21}$$

where $\vec{x}$ is vector of all error elements ($\delta q_i, \delta b_{i,x}, \delta b_{i,y}, \ldots$). For first hexapod model dimension of $\vec{x}$ is 78 and for second is 60. First three elements of $\delta\vec{\Pi}$ have different dimension than last three. For that reason search for maximal error must be divided in search for maximal end effector displacement error and search for maximal orientation error.

In both cases error can be expressed with its absolute value, absolute error in position and absolute orientation error. Since this value is computed as square root from a sum of its squared components and because square root functions are growing function, we can simplify computation by calculating square of error. Let $\delta\vec{\Pi}'$ denote error vector in position or orientation and accordingly $\underline{J}^{-1'}$ and $\underline{K}'$ denotes first or last three rows or columns of matrices $\underline{J}^{-1}$ and $\underline{K}$. Error can then be calculated as:

$$
\begin{aligned}
E^2 &= \delta\vec{\Pi}'^T \delta\vec{\Pi}' = \left(\underline{J}^{-1'}\left(\underline{K}' \cdot \vec{x}\right)\right)^T \left(\underline{J}^{-1'}\left(\underline{K}' \cdot \vec{x}\right)\right) \\
E^2 &= \vec{x}^T \underline{K}'^T \underline{J}^{-1'^T} \underline{J}^{-1'} \underline{K}' \vec{x} \qquad (22) \\
E^2 &= \vec{x}^T \underline{A}\, \vec{x}
\end{aligned}
$$

where $\underline{A}$ is an symmetric matrix with positive diagonal elements.

Assuming that each input error may vary from $-x_i$ to $+x_i$, search space can be greatly reduced to just corner points of an *n*-dimensional space. Even so, because number of corners is 2^n and n is 78 or 60, problem is still too big for exact methods of computation. Obviously some numeric method must be used which will give some result in finite time, although it may not be the maximum we are looking for.

Since error function is symmetric function, searching space can be further reduced by 50%.

Next significant search space reduction can be made with closer look to formulas (10) and (20). Error vectors ${}^P\delta\vec{a}_i$ and ${}^P\vec{d}_i$ always appear together. So do vectors $\delta\vec{b}_i$ and $\vec{c}_i$ in (10). If prior to calculation those two vectors were replaced by one, searching space would be reduced by factor 2^3 for every pair! In this way the original search space is reduced from 2^{77} to a 2^{41} for first and from 2^{59} to 2^{41} for second model.

Even that greatly reduced, the search space is still large. We used an approximate iterative numerical method very similar to coordinate axis search.

IV. EXPERIMENTAL RESULTS

The described method can be used to find maximal error in position and orientation for a single end effector pose or to evaluate possible errors in entire workspace area. We used the second approach which gives a mode to compare different hexapod models.

An error value for a single point $\vec{P}$ is calculated as average number of errors for that point with every given orientation. Those orientations $\vec{L}$ are defined with a cone as shown on Fig. 5.

Due to the long computation time we restricted error analysis to the working area only and only to errors in end effector position.

After calculating errors for each point of hexapod working area, some values are extracted to characterize the model. Those include minimum, maximum, average and standard deviation for those errors.

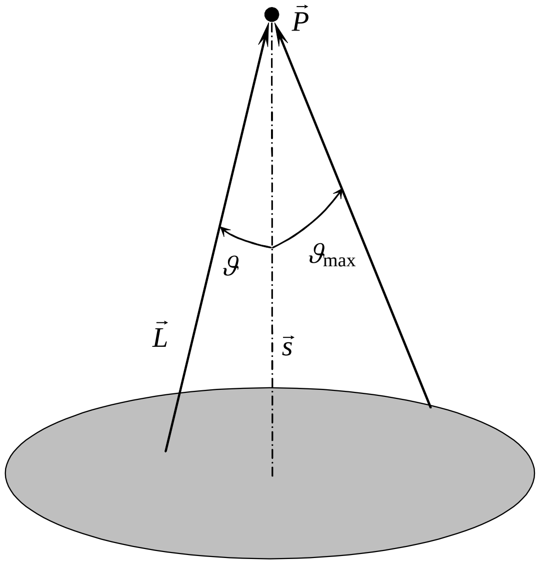

Fig. 5. Orientations used in calculations

Table 1 shows parameters for a first model of Stewart Platform based hexapod used in error analysis.

TABLE 1. Parameters for the first hexapod model

parameter	value	parameter	value
r_1	50	α	30°
r_2	25	β	30°
l_{max}	90	γ	45°
l_{min}	50	*all errors*	0.01
l_{tool}	10	*W.A.Volume*[1]	31858

Length unit isn't specified because model can be scaled with any factor and relative aspect ratios would remain the same. For example, if the unit is centimetre than errors are also in centimetres.

Error values that characterize model given by Table 1 are shown in Table 2.

TABLE 2. Error values for the first hexapod model

δr_{avg}	δr_{min}	δr_{max}	δr_{stdev}
0.2382	0.1867	0.2733	0.0167

Graphical representation as shown in Fig. 6 can tell more about how errors are spread trough workspace. Fig. 6 shows errors at cross section with plane x=0. Darker the point is the error is bigger.

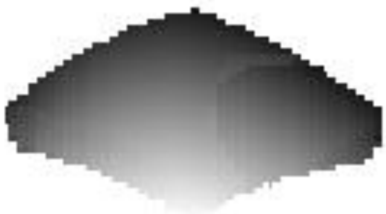

Fig. 6. Errors at cross section with plane x=0 (1)

[1] Working area volume calculated with given parameters

From central region down toward to the fixed platform, errors became smaller, while they increase towards upper working area boundaries. Abnormality in right half of Fig. 6 is a result of numeric algorithm who didn't find maximum in all points.

Parameters for second hexapod model – model with fixed strut lengths - are shown in Table 3.

TABLE 3. Parameters for the second hexapod model

parameter	**value**	**parameter**	**value**
r_{11}	75	l_{tool}	10
r_{12}	10	β	30°
h	45	γ	45°
d	10		
r_2	10	*all errors*	0.01
l	70	*W.A.Volume*	108060

Table 4 shows calculated characteristic errors for the second hexapod model found in its workspace.

TABLE 4. Error values for the second hexapod model

δr_{avg}	δr_{min}	δr_{max}	δr_{stdev}
0.3189	0.1357	0.9315	0.1444

Errors are little bigger in comparison to the first model, and show much more variation.

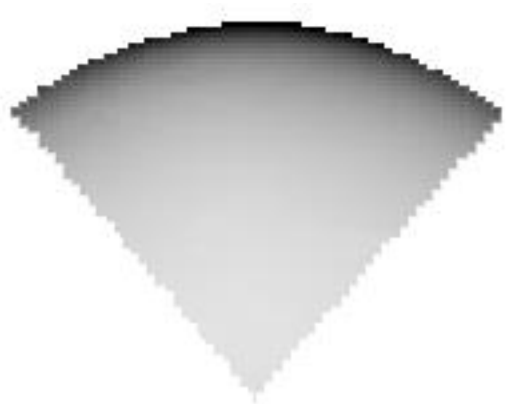

Fig. 7. Errors at cross section with plane x=0 (2)

Fig. 7 shows errors distribution at cross section with plane x=0. It also shows the shape of workspace area which is more ordinary than workspace of standard Stewart platform manipulator. Workspace volume for second hexapod is more than three times bigger but that hexapod also takes much more space due to its different construction.

As with first hexapod, error grows towards upper working area boundaries.

V. CONCLUSION

A numerical method for error analysis of two hexapod structures is presented. For input errors given with its boundary values, the method can compute maximal error at end effector, both in position and orientation.

Evaluating errors through whole workspace gives a way to compare and validate hexapods with different parameters and their resistance to errors. In this manner search for set of hexapod parameters which minimize error influence can be made.

Computationally, method is very intensive and depending on processor's speed the computation can take a very long time to finish.

Since method is based on differentiations it is applicable only for relatively small error values. It can also produce abnormally big errors which mean that end effector approaches singular area – area where an extra DOF appears that can't be controlled.

IV. ACKNOWLEDGMENT

This work was done within the research project "*Problem-Solving Environments in Engineering*", supported by the Ministry of Science and Technology of the Republic of Croatia.

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