Plain recurrent models

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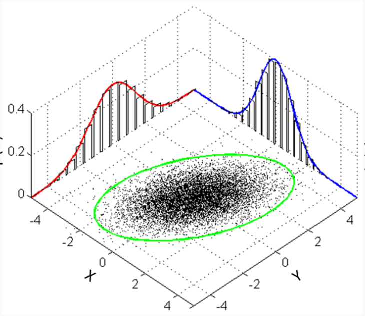
Introduction

Previously on Deep learning ...

... we have considered to model the data with:

- fixed dimensionality
- strong local covariance

images from CIFAR-10



multivariate normal distribution

Previously on Deep learning ...

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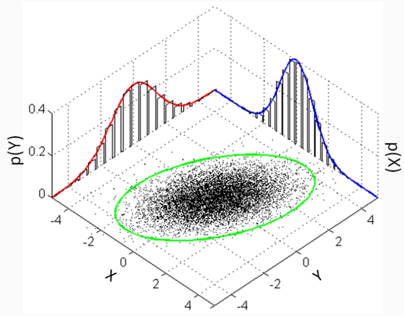


multivariate normal distribution

images from CIFAR-10

Previously on Deep learning ...

Fixed input dimensionality:

- All samples from a multivariate normal distribution (cf. Lab #1) have the same dimensionality: $x_i \in \mathbb{R}^d, \forall i$
- Each image from CIFAR-10 has the same dimensionality 32*x*32*x*3 (WxHxC)
- **How to assert:** cropping, padding, pooling, scaling (interpolation)

Local inter-dependence:

- In multi-variate normal distribution inter-dependence is defined through the distribution parameter Σ
- Neighbouring image pixels have strongly correlated RGB values (though long-range dependencies can help too)
- **How to assert:** use convolutional filters (and positional encodings in transformers)

Natural language processing

Translate the following sentence into English:

Duboko učenje je sjajno. $\rightarrow$ *Deep learning is great.*

The problem requires the following actions:

- segment the sentence (sequence of words/letters) into meaningful components (**tokens**)
- “understand” that expression “duboko učenje” does not refer to learning at great depths (eg. at the bottom of the Atlantic); instead, it refers to a branch of machine learning
- understand the meaning in Croatian without requiring a specification of the source language
- generate the English translation with the same meaning.

Textual data is strange:

- **No** fixed dimensions (sentence lengths can vary)
- **Non-local** inter-dependence patterns

Natural language processing

Translate the following sentence into Vietnamese:

最寄りのインタネットカフェはどこですか

$\rightarrow$ *Chờ internet ở đâu*

$\rightarrow$ *Where is the nearest internet shop*

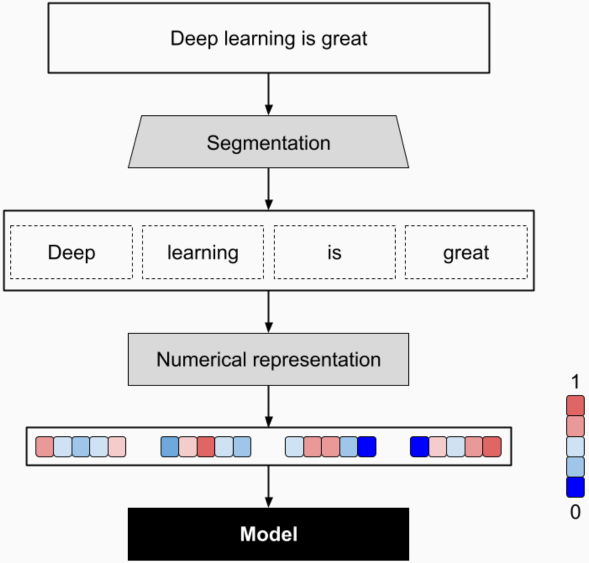
For computers, the language looks about the same as Japanese (but only for us who do not speak Japanese).

The meaning of words and similarity of language units are unknown.

Before processing the text with machine learning algorithms, we must represent words with vectorized representations.

Data representation

Natural language processing



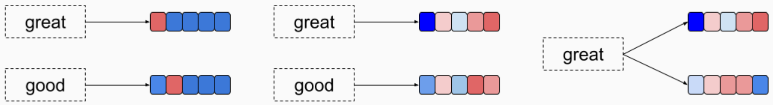
This lecture

Text segmentation:

- we can segment text in **words**, letters and subwords[1]

Numerical representation:

- associate each word with a dense vector
- alternatives: one-hot representations (*bag-of-words*), multiprototype representations

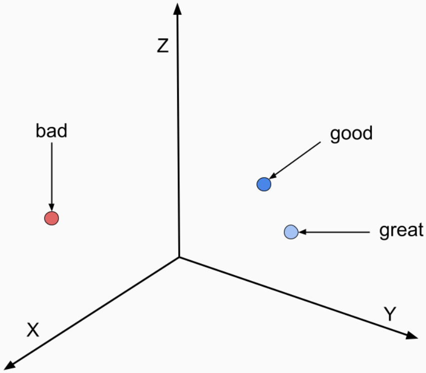


One-hot **(left)**, dense **(center)**, and multiprototype representations **(right)**.

[1]xhttps://github.com/google/sentencepiece

Dense word representations

Distance in representation space should correspond to (semantic an syntactic) similarity between words

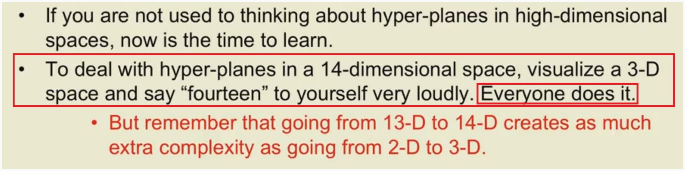


- what is the distance between one-hot word representations?
- how could we compute the distance for multiprototype representations?

Dense word representations

In practice, the dimensionality of the word representation space is considerably larger than in the figure.

We must rely on intuitions from lower-dimensional spaces although they may be deceptive.



- If you are not used to thinking about hyper-planes in high-dimensional spaces, now is the time to learn.
- To deal with hyper-planes in a 14-dimensional space, visualize a 3-D space and say “fourteen” to yourself very loudly. Everyone does it.
  - But remember that going from 13-D to 14-D creates as much extra complexity as going from 2-D to 3-D.

Lectures, “Neural Networks for Machine Learning”, Geoffrey Hinton

Word representations

We may initialize the word representations by pre-training the **tokenizer** on some surrogate (or pretext) task:

- many of these tasks predict words from local contexts on large corpora of real text (Wikipedia, Common crawl[2])
- models: word2vec[3] (CBOW, Skip-gram), GloVe[4], FastText[5]
- the training optimizes the embedding matrix that contains a vector representation for each word

In this course, we will not consider separate learning of word representations:

- instead we shall either use random initializations or rely on pre-trained embeddings.

[2] https://commoncrawl.org/

[3] https://en.wikipedia.org/wiki/Word2vec

[4] https://nlp.stanford.edu/projects/glove/

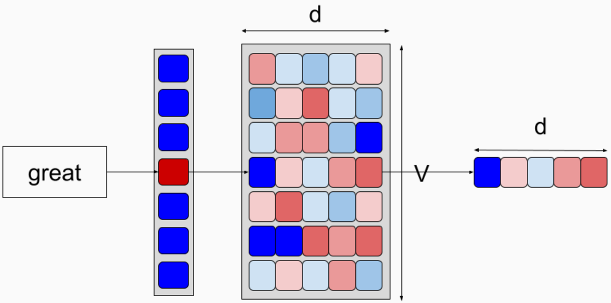
[5] https://fasttext.cc/

Embedding matrices in practice

We recover word embeddings by multiplying the learned embedding matrix $\mathbf{E} \in \mathbb{R}^{d \times V}$ with the one-hot vector of the word:

$$\mathbf{e}_i = \mathbf{E}\mathbf{x}_i \qquad \mathbf{x}_i = \underbrace{[0, \ldots, 0, \overbrace{\ 1\ }^{i}, 0, \ldots, 0]^\top}_{V}$$

Further slides will denote dense empeddings $\mathbf{e}_i$ as $\mathbf{x}_i$!



Word representations: hyper-parameters

Vocabulary size: V

- number of unique words that are recognized by the model
- it depends on available memory and word relevance[6]
- in practice we often take top V most common words from the training corpus
- the remaining words can be i) filtered our or ii) replaced with the token *<UNK>*

Word embedding dimension: d

- we often use $d = 300$ as a sensible default[7]
- in practice the choice will also depend on available memory, target performance, and pre-trained embeddings.

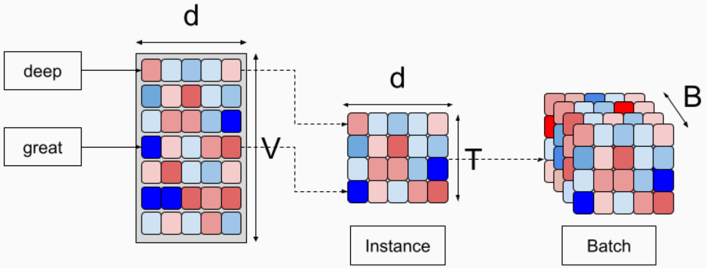
[6] rare words and words with low information content are better omitted in order to avoid performance hit.

[7] other choices may lead to under-fitting and over-fitting

Representing text

We form text representations (sentences, paragraphs, etc) by concatenating word embeddings

We denote the text length as the number of words *T* (*temporal* dimension)



This approach has to address several problems. Can you point them out?

Representing text

Problem: temporal dimension **varies**:

- sentences of a learning batch will have different lengths
- this can be addressed by zero-padding and truncation

Goal 1: model processes each word of the sentence in the same manner

- this limits the model dimensionality and reduces overfitting

Goal 2: model is sensitive to the word order

- otherwise, we would not be able to distinguish sentences with same words:
  - *Dog eats cat* vs *Cat eats dog*

We achieve both goals with **recurrent models**

- let us first look into some alternatives for addressing the variable temporal dimension.

Representing text: temporal aggregation by pooling

Express the text representation by pooling word embeddings (*eng. mean/average pooling*)

$$r_\mu(text) = \frac{1}{T}\sum_{t}^{T} x^{(t)}$$

- such aggregation loses postitional information but it does not require any **parameters**

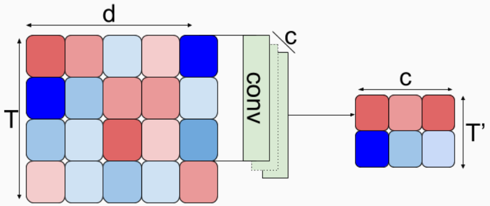
Variants of this approach: sum pooling, **weighted pooling**

$$r_w(text) = \sum_{t}^{T} w_t x^{(t)} \qquad w_t = f(x^{(t)}, \text{text})$$

- weighted pooling multiplies word embeddings with a factor that depends on the text and the word (eg. TF-IDF)

Representing text: 2D convolution

Convolutions retain information about the word ordering:



This still requires some form of pooling in order to preclude variable output

Simple convolutional architectures miss the *global* context

- this can be alleviated by increasing the depth...
- ... or by harnessing dilated convolutions or spatial pooling

Representing text: summary

Natural language processing (NLP) confronts unexpected problems

- segment the text into words, **subwords** or letters
- detect and correct typographic errors
- many ways to put the same thoughts into writing
- long-range dependencies are common

Usual assumptions:

- input text segmented into tokens
- token embeddings can be pre-trained or randomly initialized
- each word assigned to exactly *one* dense embedding

Approaches to address variable input:

- 1D convolutions and pooling layers
- recurrent modelling

Plain recurrent models

Recurrent models: motivation

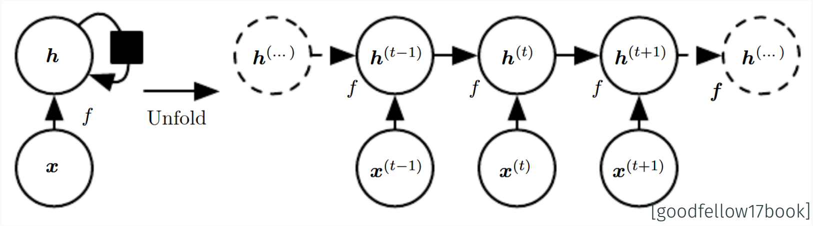
We are looking for a suitable model for sequential data:

- accomodate arbitrary sequence lengths
- number of parameters does not depend on the length
- model is aware of data ordering

Inspired by **dynamic systems**:

- **hidden state** $h^{(t)}$ uniformly updated across all inputs $x^{(t)}$:

$$h^{(t)} = f(x^{(t)}, h^{(t-1)})$$



[goodfellow17book]

Recurrent models: formulation

Recurrent models are equivariant with respect to time (location within the sequence):

$$h^{(t)} = f(x^{(t)}, h^{(t-1)})$$

A simple recurrent model involves one non-linearity for each input:

$$h^{(t)} = g(\underbrace{W_{hh}h^{(t-1)} + W_{xh}x^{(t)} + b_h}_{a^{(t)}})$$

- W_{hh}, W_{xh}, and b_h - parameters of the recurrent affine transform
- $a^{(t)}$ - recurrent linear score
- g - non-linearity (sigmoid, tanh,...)

Recurrent models: intuition

The state $h^{(t)}$ represents the **observed** portion of the sequence:

- we hope that it encodes semantics if the model is well-learned

The matrix W_{xh} projects the input into the representation space:

- we hope that this projection removes useless information from the input

The matrix W_{hh} models the evoulution of the state:

- it models the passage of time and hopefully removes the excess information from the state

$$h^{(t)} = g(W_{hh}h^{(t-1)} + W_{xh}x^{(t)} + b_h)$$

The dimensionalities of the state and the input **may differ**:

$$h \in \mathbb{R}^h \qquad x \in \mathbb{R}^d$$

Recurrent models: output

The state $h^{(t)}$ represents all observed information so far:

- besides semantics, the state must also remember complementary information
  - eg. ambiguous words or references to people or locations
- some of these pieces of information may be **irrelevant** for prediction!

The *output layer* projects the state into the output space:

$$o^{(t)} = W_{hy} h^{(t)} + b_o$$

- the vector $o^{(t)}$ contains logits with predictions at time t
- this steps *filters* unimportant information
- **[!!]** Pytorch RNN cell does not contain the output projection – you have to add it explicitly.

Recurrent models: formulation

A simple recurrent model is defined by the following equations:

1. update of the hidden state
$$h^{(t)} = tanh(W_{hh}h^{(t-1)} + W_{xh}x^{(t)} + b_h)$$
2. projection into output space
$$o^{(t)} = W_{hy}h^{(t)} + b_o$$

In practice, we often activate the state with tanh activation:

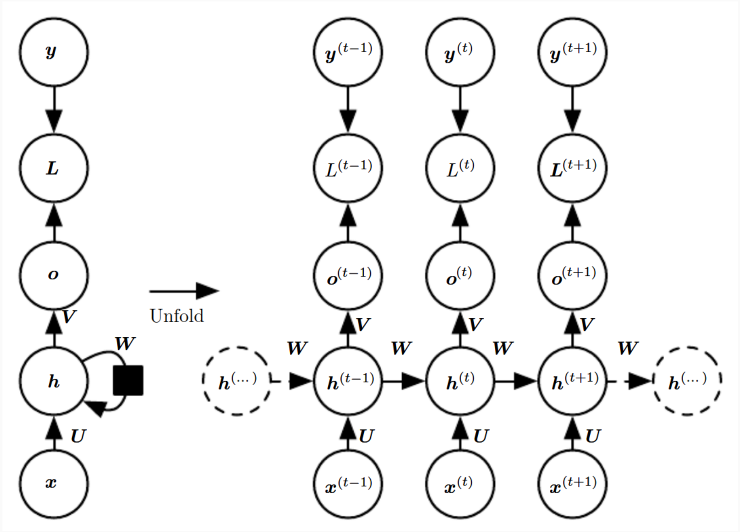
- other options are the sigmoid and the hinge.

Parameter dimensions: $W_{hh} \in \mathbb{R}^{h\times h}$, $W_{xh} \in \mathbb{R}^{h\times d}$, $W_{hy} \in \mathbb{R}^{y\times h}$

- d and y denote the input and the output dimensionality

[!!] Textbook notation: $W := W_{hh}$, $U := W_{xh}$, $V := W_{hy}$, $b := b_h$, $c := b_o$

Recurrent models: visualization



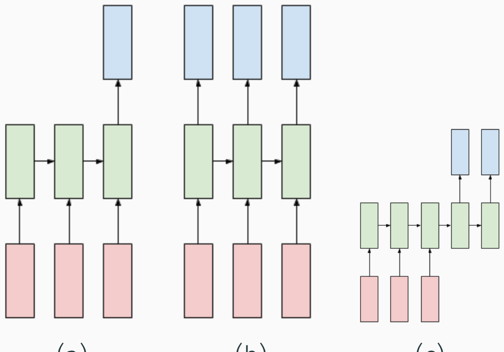
Unfolded recurrent model with inputs $x^{(t)}$, loss $L^{(t)}$ and outputs $o^{(t)}$.

Adapting the model to the task

A recurrent model can generate output at each time step – but do we need that?

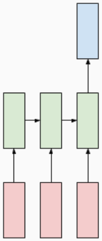
- output configuration of our models depend on particular task!

Try to think of problems which require only one output (a), output in each input (b), and variable number of outputs, but more than one (c).



NLP Tasks: sequence-wide prediction

Sequence classification: the basic NLP problem.



Sequence classification

The task consists in determining one target variable for the entire input sequence

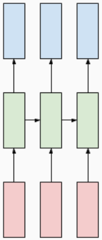
Examples of classification problems:

- sentiment analysis
- document categorization
- determining a music genre

The output layer receives only **the last** latent state.

NLP Tasks: per-token prediction

Sequence ”labeling: requires a prediction in each input element:


Sequence ”labeling”

the task involves a sequence of targets; each output corresponds to exactly one input

Examples of problems:

- part-of-speech ”tagging”
- named entity recognition
- extractive text summarization
- segmentation of text in video, ...

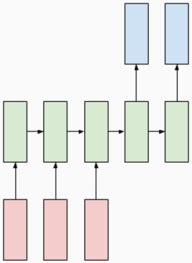
Predictions do not have to be synchronized with the inputs (there can exist a small temporal offset), but this is often the case in practice.

We use quotes since the word *labeling* suggests human annotation

- better term: dense prediction

NLP Tasks: sequence-to-sequence translation

Sequence-to-sequence (*seq2seq*) tasks involve more general forms of generating sequences from sequences.


Sequence-to-sequence

The task is to determine the target sequence of unknown length given the input

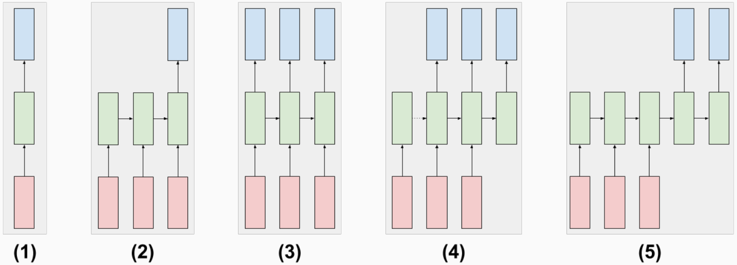
Examples:

- machine translation
- abstractive tex summarization

Consider potential problems of this task:

- how to start the translation?
- should we always generate the most probable word?
- how to decide when to conclude the translation?

NLP Tasks: overview



Directions for problem classification:

- whenever the number of outputs equals the number of inputs, the problem coresponds to token-level (dense) prediction regardless of whether the offset exists (4) or not (3)
- sequence-to-sequence problems (5) are often decomposed into text summarization (where we do not predict any new information) and sequence generation (which generates the entire output sequence)

Analysis: Sequence classification

Sentiment analysis

Classification problem: to determine the sentiment of the input text

- binary formulation: positive/negative
- categoric formulation: star rating [1,10]
  - this could also be formulated as regression
- example datasets: IMDB[8], YELP[9]
- a positive example from IMDB:

```
... was the story that blew me away. hurray for Takahisa
```

Our analysis assumes binary formulation and neglects (considerable) pre-processing issues.

[8] https://ai.stanford.edu/~amaas/data/sentiment/
[9] https://www.yelp.com/dataset/

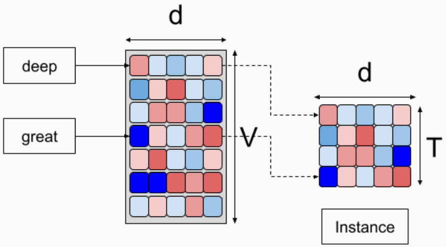
Sentiment analysis: preliminaries

Target variable:

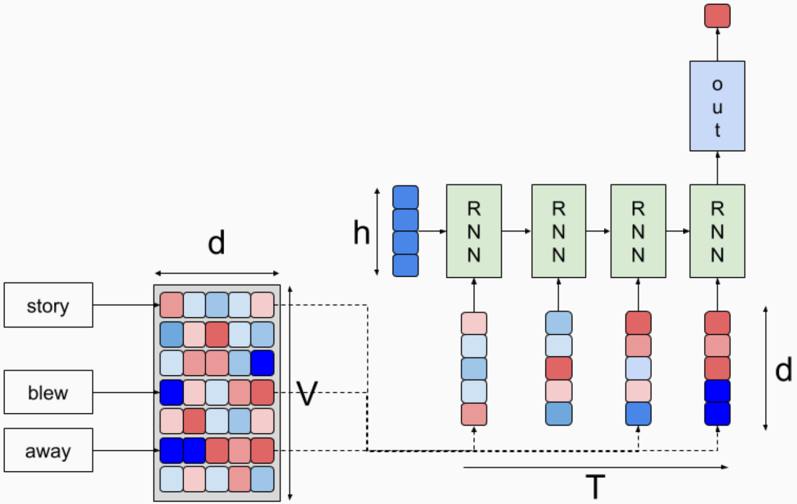
- sentiment variables: {positive, negative}
- we represent them as indices: $\{0, 1\}$

Input variable:

- a sequence of tokens that represent words, tokens or letters
- we must select the size of the input vocabulary
- we map words onto vectors of the embedding matrix (or <UNK>)



Sentiment analysis: top-level sketch



Sentiment analysis: pseudocode

Algorithm 1: Plain RNN for sequence classification

```
Single output RNN ($X, W_{hh}, W_{xh}, W_{hy}, b_h, b_o$)
    inputs : A sequence of vectors $X$, parameters $W_{\dots}$, $b_{\dots}$
    output: A logit $\hat{y}$
    $h_t \leftarrow zero\_init$;
    foreach $x$ in $X$ do
        $a_t \leftarrow W_{hh}h_t + W_{xh}x + b_h$;
        $h_t \leftarrow tanh(a_t)$;
    end
    $o \leftarrow W_{hy}h_t + b_y$;
    return $o$;
```

We can adapt this algorithm towards dense prediction by generating elements of the output vector within the loop.

Analysis: dense prediction

Part-of-speech ”tagging” (better: recognition)

Dense prediction along the sequence: predict a POS class for each word

- multi-class formulation: noun, adjective, verb, ...

Datasets: Penn Treebank (PTB) (behind paywall), Universal Dependencies[10] (UD)

- PTB taxonomy: `https://www.ling.upenn.edu/courses/Fall_2003/ling001/penn_treebank_pos.html`

Training example, U-POS labels and POS labels:

```
I    do  n't  think it   matters.
PRON AUX PART VERB  PRON VERB   .
PRP  VBP RP   VB    PRP  VBZ    .
```

[10] `https://universaldependencies.org/`

Part-of-speech recognition: preliminaries

Target variables:

- part-of-speech class: $\{ADP, PROPN, ...\}$
- we leverage class indices: $\{1, \ldots, Y\}$

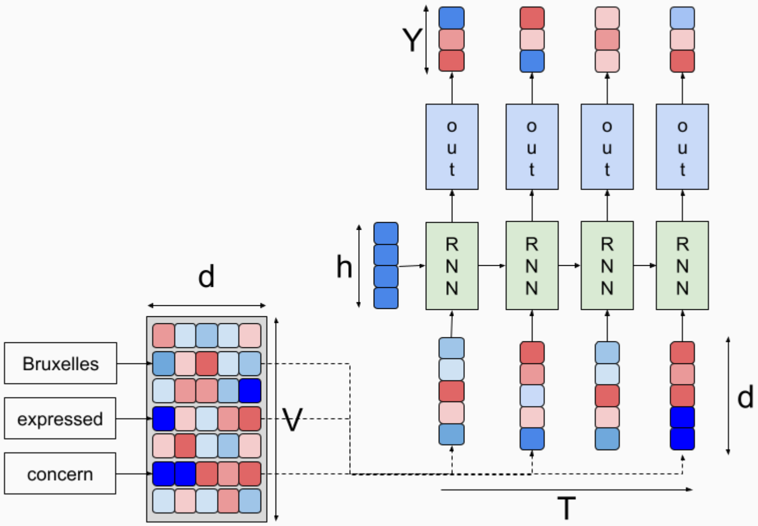
Input variables:

- we must choose the size of the vocabulary **(!!)**
- we map words onto vectors of the embedding matrix

Very similar to sequence classification:

- important difference: target dimensionality
- we have to use dense loss instead of sequence-wide loss

Part-of-speech tagging: top-level sketch



Part-of-speech tagging: pseudocode

Algorithm 2: RNN for multi-class dense prediction

```
Dense sequence prediction RNN ($X, W_{hh}, W_{xh}, W_{hy}, b_h, b_o$)
    inputs : A sequence of vectors $X$, parameters $W_{...}$, $b_{...}$
    output: A sequence of logits $\hat{y}$
    $h_t \leftarrow zero\_init$;
    $o \leftarrow [\,]$;
    foreach $x$ in $X$ do
        $a_t \leftarrow W_{hh}h_t + W_{xh}x + b_h$;
        $h_t \leftarrow tanh(a_t)$;
        $o_t \leftarrow W_{hy}h_t + b_y$;
        $o.append(o_t)$
    end
    return $o$;
```

Summary

Recurrent model acts as a dynamical system:

- input information is iteratively built into the latent state
- the latent state is iteratively updated with respect to the old state and current input.

The state (most often) does not contain predictions:

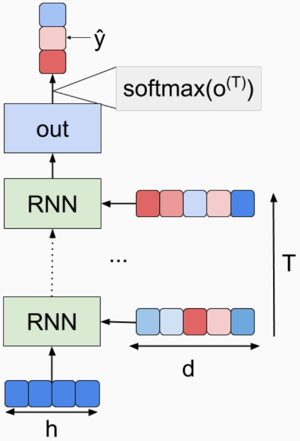
- predictions are formed as a learned projection of the state
- the actual task determines the number of projections (only one, per-input or variable)
- if there are multiple projections, their parameters are **shared**.

The three main tasks are **sequence classification**, **dense prediction** and **sequence-to-sequence translation**.

Training recurrent models

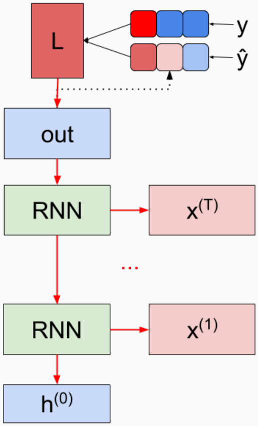
Introduction

Recurrent models can be unfold into feed-forward models with distinct inputs and shared parameters:

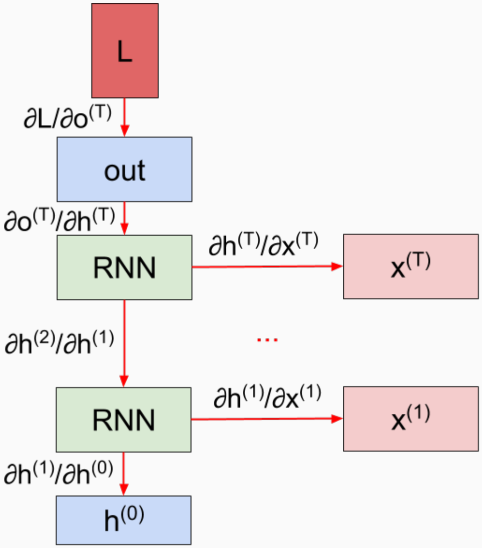


Example: sequence classification

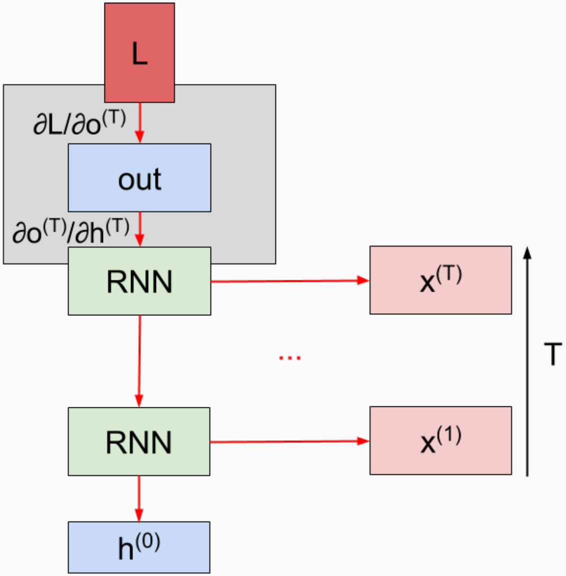
A model for sequence-wide prediction can be trained with standard backprop:



Example: sequence classification (2)


L
$\partial L/\partial o^{(T)}$
out
$\partial o^{(T)}/\partial h^{(T)}$
RNN
$\partial h^{(T)}/\partial x^{(T)}$
$x^{(T)}$
$\partial h^{(2)}/\partial h^{(1)}$
...
RNN
$\partial h^{(1)}/\partial x^{(1)}$
$x^{(1)}$
$\partial h^{(1)}/\partial h^{(0)}$
$h^{(0)}$

RNN: backprop through the output layer



RNN: backprop through the output layer (2)

We obtain predictions by activating the outputs (reminder):

$$\hat{y} = \mathrm{softmax}(o^{(T)})$$

We assume the standard cross-entropy loss:

$$\mathbb{L} = -\sum_{j} y_j \cdot \log \hat{y}_j$$

Gradients wrt output activations:

$$\frac{\partial \mathbb{L}}{\partial o^{(T)}} = (\underbrace{softmax(\mathsf{o}^{(\mathsf{T})})}_{\hat{y}} - y)^{\top}$$

RNN: backprop through the output layer (3)

Equation of the output projection (reminder):

$$o^{(T)} = W_{hy} h^{(T)} + b_y$$

Gradients wrt parameters W_{hy} and b_y:

$$\frac{\partial \mathbb{L}}{\partial W_{hy}} = \frac{\partial \mathbb{L}}{\partial o^{(T)}} \frac{\partial o^{(T)}}{\partial W_{hy}} = \ldots = (\hat{y} - y) \cdot (h^{(T)})^\top$$

$$\frac{\partial \mathbb{L}}{\partial b_y} = \frac{\partial \mathbb{L}}{\partial o^{(T)}} \frac{\partial o^{(T)}}{\partial b_y} = (\hat{y} - y)^\top$$

Gradient with respect to the last state:

$$\frac{\partial \mathbb{L}}{\partial h^{(T)}} = \frac{\partial \mathbb{L}}{\partial o^{(T)}} \frac{\partial o^{(T)}}{\partial h^{(T)}} = (\hat{y} - y)^\top W_{hy}$$

RNN: backprop through the state update

The gradient of the loss propagates towards earlier timesteps:

$$h^{(t)} = f(h^{(t-1)}, \dots, h^{(0)})$$

Backprop at time $t = T$ (last timestep, reminder):

$$\frac{\partial \mathbb{L}}{\partial h^{(t)}} = \frac{\partial \mathbb{L}}{\partial o^{(T)}} \frac{\partial o^{(T)}}{\partial h^{(T)}} = (\hat{y} - y)^\top W_{hy}$$

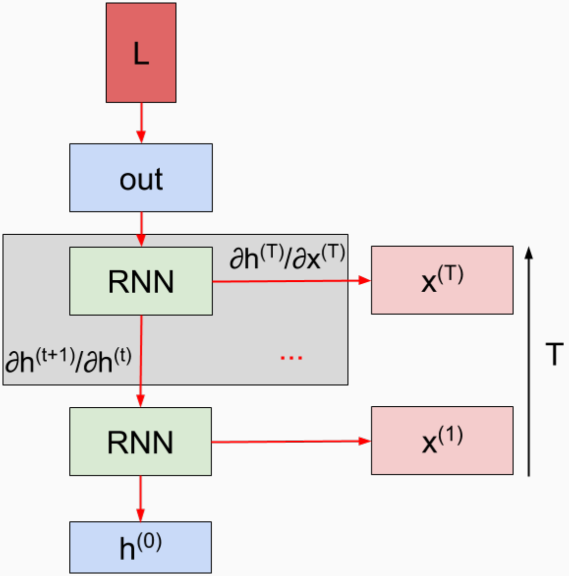
Backprop for the remaining time-steps $t < T$:

$$\frac{\partial \mathbb{L}}{\partial h^{(t)}} = \frac{\partial \mathbb{L}}{\partial h^{(T)}} \frac{\partial h^{(T)}}{\partial h^{(T-1)}} \cdots \frac{\partial h^{(t+1)}}{\partial h^{(t)}}$$

In general, two kinds of gradients may arrive to a hidden state:

1. gradients from the **corresponding** prediction
   (only in the dense prediction case)
2. gradients from the **future** hidden states

RNN: backprop through the state update (2)



L
out
RNN
$\partial h^{(T)}/\partial x^{(T)}$
$x^{(T)}$
$\partial h^{(t+1)}/\partial h^{(t)}$
...
T
RNN
$x^{(1)}$
$h^{(0)}$

RNN: backprop through the state update (3)

Equation of the RNN state update (reminder):

$$h^{(t)} = tanh(\underbrace{W_{hh}h^{(t-1)} + W_{xh}x^{(t)} + b_h}_{a^{(t)}})$$

We require the derivative of the hyperbolic tangent:

$$tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\frac{dtanh(x)}{dx} = 1 - tanh^2(x)$$

Hyperbolic tangent is a sigmoid “stretched” over $[-1, 1]$

$$tanh(x) = 2\sigma(2x) - 1$$

RNN: backprop through the state update (4)

Equation of the RNN state update (reminder):

$$h^{(t)} = tanh(\underbrace{W_{hh}h^{(t-1)} + W_{xh}x^{(t)} + b_h}_{a^{(t)}})$$

Gradient wrt the pre-activation $a^{(t)}$:

$$\frac{\partial \mathbb{L}}{\partial a^{(t)}} = \frac{\partial \mathbb{L}}{\partial h^{(t)}} \frac{\partial h^{(t)}}{\partial a^{(t)}} = \frac{\partial \mathbb{L}}{\partial h^{(t)}} \frac{\partial tanh}{\partial a^{(t)}} = ... = \frac{\partial \mathbb{L}}{\partial h^{(t)}} \odot (1 - h^{(t)^2})$$

Gradients wrt the RNN cell parameters (W_{hh}, W_{xh}, b_h):

$$\frac{\partial \mathbb{L}}{\partial W_{hh}} = \frac{\partial \mathbb{L}}{\partial a^{(t)}} \frac{\partial a^{(t)}}{\partial W_{hh}} = ... = \left(\frac{\partial \mathbb{L}}{\partial a^{(t)}}\right)^\top \left(h^{(t-1)}\right)^\top$$

$$\frac{\partial \mathbb{L}}{\partial W_{xh}} = \frac{\partial \mathbb{L}}{\partial a^{(t)}} \frac{\partial a^{(t)}}{\partial W_{xh}} = ... = \left(\frac{\partial \mathbb{L}}{\partial a^{(t)}}\right)^\top \left(x^{(t)}\right)^\top$$

$$\frac{\partial \mathbb{L}}{\partial b_h} = \frac{\partial \mathbb{L}}{\partial a^{(t)}} \frac{\partial a^{(t)}}{\partial b_h} = \frac{\partial \mathbb{L}}{\partial a^{(t)}}$$

RNN: backprop through the state update (5)

Equation of the RNN state update (reminder):

$$h^{(t)} = tanh(\underbrace{W_{hh}h^{(t-1)} + W_{xh}x^{(t)} + b_h}_{a^{(t)}})$$

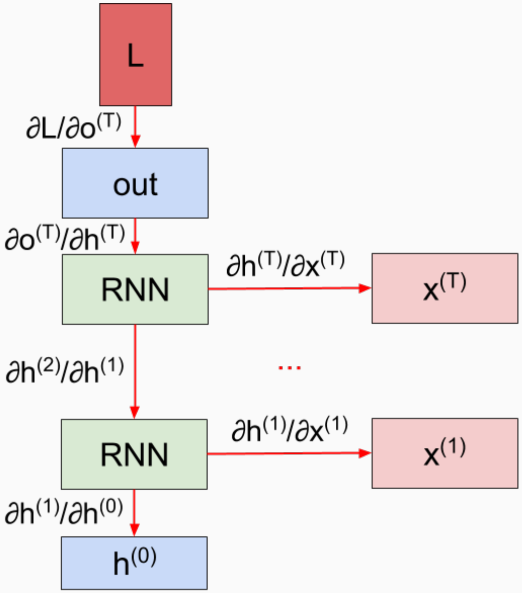
Gradient wrt the previous hidden state:

$$\frac{\partial \mathbb{L}}{\partial h^{(t-1)}} = \frac{\partial \mathbb{L}}{\partial a^{(t)}} \frac{\partial a^{(t)}}{\partial h^{(t-1)}} = \frac{\partial \mathbb{L}}{\partial a^{(t)}} W_{hh}$$

Gradient wrt input (think why!):

$$\frac{\partial \mathbb{L}}{\partial x^{(t)}} = \frac{\partial \mathbb{L}}{\partial a^{(t)}} \frac{\partial a^{(t)}}{\partial x^{(t)}} = \frac{\partial \mathbb{L}}{\partial a^{(t)}} W_{xh}$$

RNN: backprop through sequence classification, summary



RNN: backprop through sequence classification, details

Backprop reaches the state update parameters in each hidden state:

$$\underbrace{\frac{\partial \mathbb{L}}{\partial W_{hh}}\bigg|_{a^{(t)}} = \frac{\partial \mathbb{L}}{\partial a^{(t)}} \frac{\partial a^{(t)}}{\partial W_{hh}} = \left(\frac{\partial \mathbb{L}}{\partial a^{(t)}}\right)^{\top} \left(h^{(t-1)}\right)^{\top}}_{\forall t \in \{1,2,\ldots,T\}}$$

These gradient **accumulate** through time:

$$\frac{\partial \mathbb{L}}{\partial W_{hh}} = \sum_{t=1}^{\top} \frac{\partial \mathbb{L}}{\partial W_{hh}}\bigg|_{a^{(t)}} = \sum_{t=1}^{\top} \left(\frac{\partial \mathbb{L}^{(t)}}{\partial a^{(t)}}\right)^{\top} \left(h^{(t-1)}\right)^{\top}$$

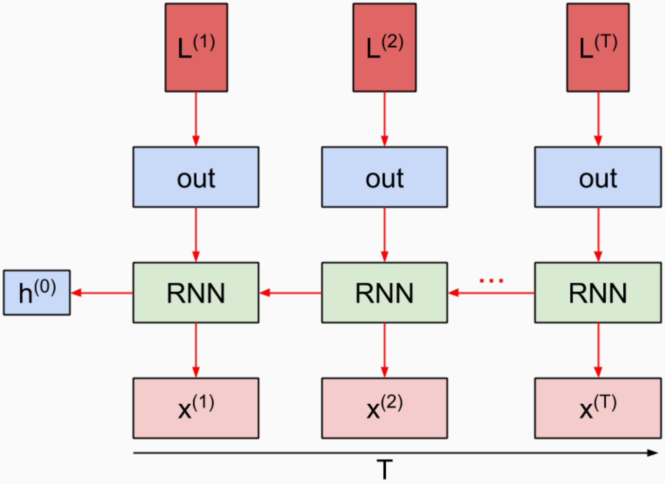
The optimization step uses the accumulated gradient.

Motivation for recovering gradients wrt inputs ($x^{(t)}$):

1. learning (fine-tuning) the word embedding matrix
2. the input may correspond to the output of another recurrent module.

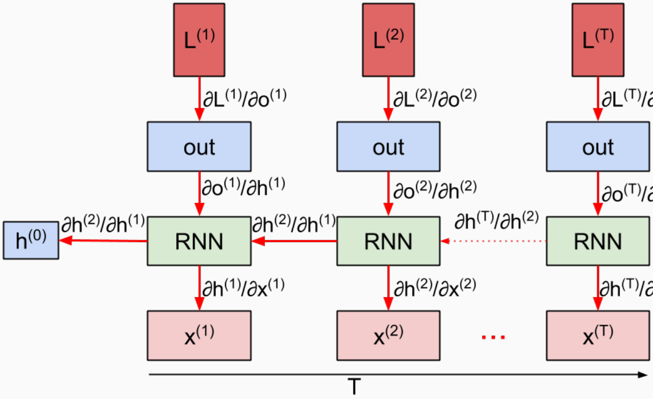
Dense prediction loss

Dense prediction involves the loss in **each** time step



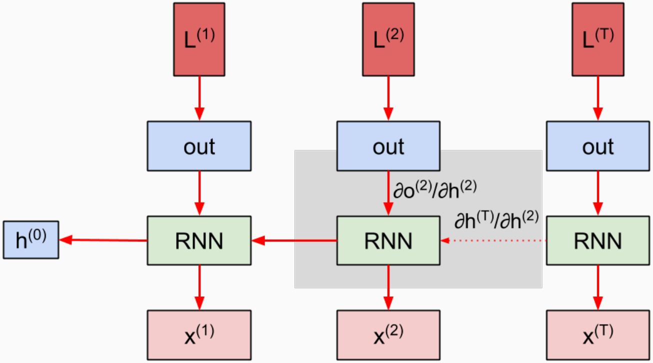
Backprop through dense prediction

In this case, backprop has to accumulate the loss terms from all future time steps:


$L^{(1)}$
$L^{(2)}$
$L^{(T)}$
$\partial L^{(1)}/\partial o^{(1)}$
$\partial L^{(2)}/\partial o^{(2)}$
$\partial L^{(T)}/\partial o^{(T)}$
out
out
out
$\partial o^{(1)}/\partial h^{(1)}$
$\partial o^{(2)}/\partial h^{(2)}$
$\partial o^{(T)}/\partial h^{(T)}$
$h^{(0)}$
$\partial h^{(2)}/\partial h^{(1)}$
RNN
$\partial h^{(2)}/\partial h^{(1)}$
RNN
$\partial h^{(T)}/\partial h^{(2)}$
RNN
$\partial h^{(1)}/\partial x^{(1)}$
$\partial h^{(2)}/\partial x^{(2)}$
$\partial h^{(T)}/\partial x^{(T)}$
$x^{(1)}$
$x^{(2)}$
...
$x^{(T)}$
T

Backprop through dense prediction: gradients (1)

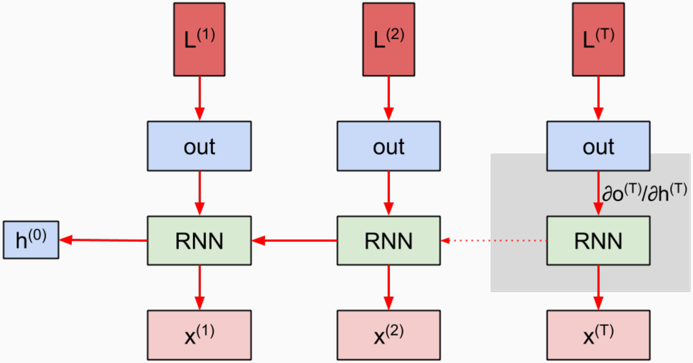
State update is affected both by the current loss (vertical dependency) and by all future losses (horizontal dependency):



Backprop through dense prediction: gradients (2)

State update is affected both by the current loss (vertical dependency) and by all future losses (horizontal dependency):

- … except at the last time-step where we do not have any contribution from the future



Backprop through dense prediction: gradients (3)

The loss corresponds to a sum of temporal components $\mathbb{L} = \sum_t \mathbb{L}^{(t)}$

- hand-coded gradients require a special care in $\frac{\partial \mathbb{L}}{\partial h^{(t)}}$

The last time-step ($t = T$) is the same as in the sequence-wide case:

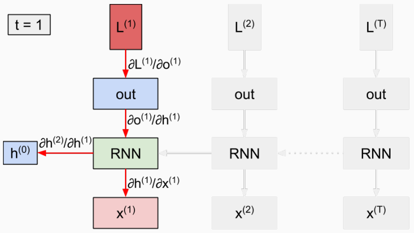
$$\frac{\partial \mathbb{L}}{\partial h^{(T)}} = \frac{\partial \mathbb{L}^{(T)}}{\partial h^{(T)}} = \frac{\partial \mathbb{L}^{(T)}}{\partial o^{(T)}} \frac{\partial o^{(T)}}{\partial h^{(T)}} = \frac{\partial \mathbb{L}^{(T)}}{\partial o^{(T)}} W_{hy}$$

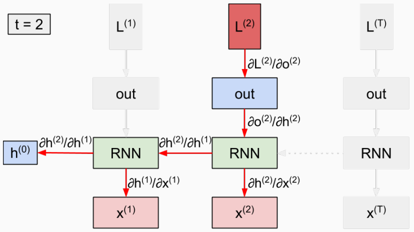
Special considerations for $t < T$ (the remaining time-steps):

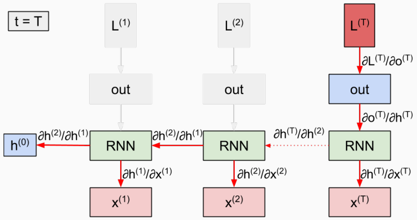
$$\frac{\partial \mathbb{L}}{\partial h^{(t)}} = \underbrace{\frac{\partial \mathbb{L}^{(t)}}{\partial h^{(t)}}}_{\text{current loss}} + \underbrace{\frac{\partial \mathbb{L}^{(t^*>t)}}{\partial h^{(t)}}}_{\text{future loss}}$$

$$\frac{\partial \mathbb{L}^{(t^*>t)}}{\partial h^{(t)}} = \sum_{t^*>t}^{T} \frac{\partial \mathbb{L}^{(t^*)}}{\partial h^{(t)}} = \sum_{t^*>t}^{T} \frac{\partial \mathbb{L}^{(t^*)}}{\partial h^{(t^*)}} \cdots \frac{\partial h^{(t+1)}}{\partial h^{(t)}}$$

Backprop through dense prediction: intuition







Each RNN cell receives gradients from the above (loss at time t) and the right (loss from the future)

In pytorch, we can perform either:

- T backward passes (per-time-step, with `retain_graph=True`)
- 1 backward pass through the total loss.

Backprop through dense prediction: summary

Backprop through recurrent models is often referred to as backpropagation through time (BPTT):

- conceptually, BPTT is equivalent to the standard backprop through a fully-connected model
- dense prediction can be interpreted as a jointly optimized multi-task classification problem
- RNN cell can be viewed as a layer of a plain fully-connected model

Due to shared parameters and gradient accumulation, training can become **unstable**:

- high incidence of vanishing and exploding gradients
- the presented basic formulation is seldom used in practice

Practical advice

Dimensionality of the latent state h:

- as large as possible, other hiperparams may be more important
- model depth may also be more important (next lecture)

Sequence length T:

- plain RNNs have **very** short-term memory ($T \approx 20$ for text)
- we often clip samples in order to enforce some maximal length
- the threshold depends on the data and the task

Sensible defaults: Adam, gradient clipping (even with LSTMs)

RNN alternatives **exist**:

- SVM for simple text classification: `https://github.com/mesnilgr/nbsvm`
- **attention** may be all you need
- sound: (dilated) convolutions, attention

Questions?

Additional reading

Reading list (besides the textbook):

1. Peter's notes: Implementing a NN / RNN from scratch `http://peterroelants.github.io/`
2. Andrej Karpathy: Unreasonable Effectiveness of Recurrent Neural Networks `http://karpathy.github.io/2015/05/21/rnn-effectiveness/`
3. Cristopher Olah: Understanding LSTM's `http://colah.github.io/posts/2015-08-Understanding-LSTMs/`
4. CS224d: Deep Learning for Natural Language Processing `http://cs224d.stanford.edu/syllabus.html`