Gradient of B-splines in Volume Rendering

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Abstract — **This paper deals with surface reconstruction and the gradient reconstruction in the volume rendering. In the volume rendering procedure, reconstruction according discrete set of samples is required. Due to the reconstruction procedure alias artifacts, in the final image, could not be neglected. In this paper we focus our attention on the gradient reconstruction based on the surface reconstruction. For the surface reconstruction we use the cubic B-splines and for the gradient reconstruction corresponding derivative.**

If the noise is present in the input data the approximation B-spline is used, and the interpolation B-spline is used for the input signal without the noise. The shading procedure requires normal estimation. Two approaches are used for normal estimation. The classic approach for normal estimation is central difference calculation, and we propose derivative calculation of the reconstruction B-spline function. We show that calculation of the normal vector has important influence on the alias artifacts in the result.

I. INTRODUCTION

In the volume visualization input data are sampled on the regular rectilinear grid. Computer tomography (CT), magnetic resonance (MR), ultrasound slices, and numerically generated data are examples of data acquisition for volume visualization. Volume visualization enables visual insight in the object. This noninvasive technique is important for medical purposes, fluid visualization, engine visualization and numerous other applications. It is very important to reduce the errors introduced by the visualization procedure and to render the reconstructed object as accurately as possible. The alias artifacts in the result can cause incorrect interpretation of the object.

The volume rendering [4] is one of the visualization techniques. In the volume rendering rays are cast from the viewer through the projection plane in the volume element space. Along the rays, at arbitrary positions in the volume element space, reconstruction is required. The reconstruction is done using the samples that are positioned at the regular rectilinear grid. According to reconstructed value, intersection of the ray with the object surface is defined. At that intersection, illumination is calculated. For the illumination calculation, normal vector on the surface is required. Since the surface of the object is implicitly defined, definition of normal vector is not straightforward.

For the reconstruction purpose it is best to find the continuous function $f(x, y, z)$ according to the given set of samples. When continuous function is defined, one can easily find the value at the arbitrary position in the three-dimensional space, at the resampling points as well as derivatives at those points. But, the intersection point of the ray with the surface is still implicitly defined.

The normal vector of the tangent plane on the B-spline surface $\mathbf{x}(u,v) = [x(u,v) \quad y(u,v) \quad z(u,v)]^T$ in the parametric form coincides with the normal to the surface at $\mathbf{x}$. The normalized normal is:

$$n = \frac{x_u \times x_v}{\| x_u \times x_v \|}, \tag{1}$$

where $\times$ is cross product and x_u, x_v are partial derivatives [2]. The tangent is defined by $\mathbf{t} = \mathbf{x}'$, and the main normal $\mathbf{m}$ as $\mathbf{t}' = \kappa\mathbf{m}$. The surface of the object in the volume element space is implicitly defined by the threshold value f_0. The frontier of $f(\mathbf{x}) < f_0$ defines surface, where the $\mathbf{x}$ is any point in the volume element space. At these circumstances we approximate the normal vector on the surface with the gradient at that point.

The reconstruction problem appears in various algorithms in computer graphics e. g. texture mapping image rotation, ray tracing, scan conversion. The derivative calculation is useful for edge detection in image processing. The basic ideas and experience in volume reconstruction are also applicable on similar problems in other fields and vice versa.

Much work has been done towards the design of reconstruction filters and error characterization [5], [6], and [9]. Simple approaches are nearest neighbor and trilinear interpolation, but continuity of the reconstructed function is only C^0 and C^1 respectively. Better approaches for reconstruction are cubic spline, e. g. BC-splines introduced by Mitchell and Netravali [8], Catmull-Rom spline and approximation and interpolation B-splines [11], [12].

Bentum [1] analyses responses of the gradient filters in frequency domain, but he did not consider interpolation or least squares B-splines.

The high quality volume rendering of the isosurface requires continuous and continuously differentiable model of volumetric discrete and regular data. P. Thévenaz and M. Unser [10] show that quadratic B-spline is the shortest support function that maximizes order of approximation that satisfies the continuity requirements. They also propose several preprocessing steps that significantly accelerate the rendering.

Our approach is based on the cubic B-spline [3], [7] and we show the artifacts that appear due to the approximations in normal estimation.

II. B-SPLINES

In computer graphics B-splines are the most important for curve and surface interpolation and approximation [2]. Non-uniform rational B-splines (NURBS) enable conic section representation and more control, but for many purposes simple uniform non-rational B-splines are sufficient. Uniformly spaced samples in the volume element space are suitable for periodic uniform B-spline interpolation and approximation.

The original definition of the B-splines is appropriate for non-uniform representation, but for the uniform case, representation of the B-splines as convolution form in signal processing is more intuitive and useful [11].

A. Approximation B-spline

For a given set of $(n+1)$ control points $\mathbf{r}_i$, the approximation B-spline curve $\mathbf{p}(t)$ is:

$$\mathbf{p}(t) = \sum_{i=0}^{n} \mathbf{r}_i \, \mathrm{N}_{i,k}(t), \tag{2}$$

where $\mathrm{N}_{i,\,k}$ are the basis or blending functions of degree k. For the sequence of the points $\mathbf{r}_i$, this formula defines the continuous curve, where $\mathbf{p}(t)$ is a point on the curve, for a given parameter t. For uniformly spaces knots, the basis functions that multiply each control point become the same, but shifted to the position of corresponding point. At this circumstances convolution form is more appropriate. Without lost of generality, we focus our attention on the cubic case. We can represent the equation (2) in signal processing terms as circular convolution:

$$g^3(x) = y * \beta = \sum_{k=0}^{n} y(k)\beta^3(x-k). \tag{3}$$

In the signal processing, input is usually one dimensional $y(k)$, but it could be extend to higher dimensions. $\beta^3(x)$ is the filter kernel or the cubic B-spline basis function.

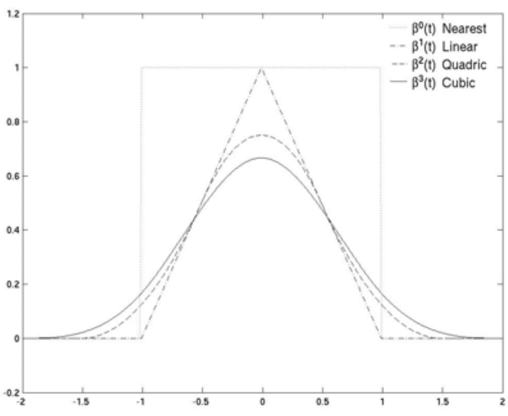

Figure 1. B-splines of the degree 0 to 3.

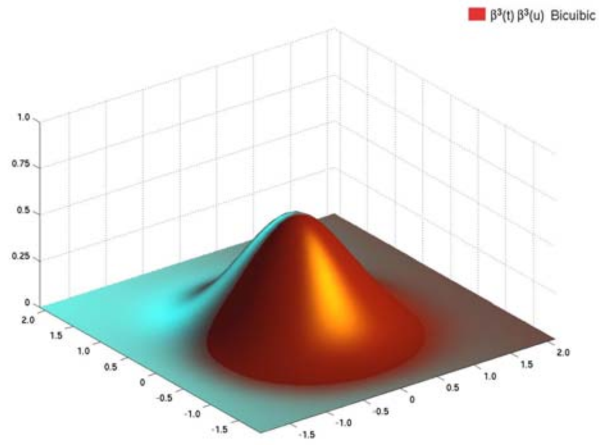

Figure 2. Bicubic B-spline filter kernel.

For the cubic case, the filter kernel is [11]:

$$\beta^3(x) = \underbrace{\beta^0 * \ldots * \beta^0(x)}_{4\ times} \tag{4}$$

where $\beta^0(x)$ is a central normalized rectangular pulse. The B-spline basis functions $\beta^0(x)$, $\beta^1(x)$, $\beta^2(x)$, and $\beta^3(x)$ are shown in Figure 6. The support width of the cubic B-spline basis function is $[-2,2]$ and cubic B-spline is:

$$\beta^3(x) = \begin{cases} \left(3x^3 - 6x^2 + 4\right)/6 & 0 \le |x| \le 1 \\ (2-x)^3/6 & 1 \le |x| \le 2 \\ 0 & 2 < |x| \end{cases} \tag{5}$$

For the two-dimensional mesh of uniformly spaced $m \times n$ control points. Two-dimensional circular convolution with the bicubic B-spline filter kernel is:

$$g^3(x,\ y) = \sum_{k=0}^{n}\sum_{l=0}^{m} y(k,l)\beta^3(x-k, y-l) \tag{6}$$

We show in Figure 2. bicubic B-spline filter kernel. For the volume rendering filter kernel is expanded to one dimension more.

B. Interpolation B-spline

The approximation B-spline is appropriate for the reconstruction if noise is present in the input data, because lowpass filtering is included in this spline. The interpolation B-spline produces function $f(x,y,z)$ that interpolates the input data, but it requires highpass prefiltering. The prefiltering is preprocessing step and it could be implemented very efficiently [12]. Frequency response of the highpass prefilter for the cubic B-spline interpolation is:

$$H(\omega) = \frac{3}{2+\cos(\omega)} \tag{7}$$

Figure 3. shows twodimensional frequency response of the cubic B-spline highpass prefilter.

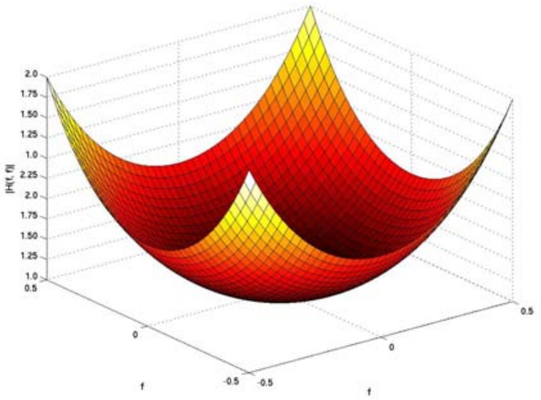

Figure 3. Two-dimensional frequency response of the highpass prefilter for the cubic B-spline interpolation.

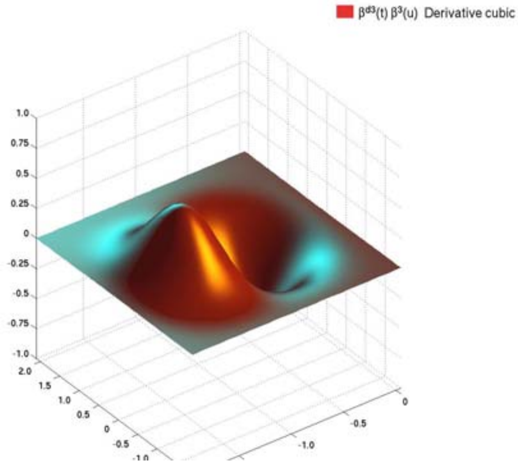

Figure 5. Partial derivative of the bicubic B-spline filter kernel.

III. NORMAL ESTIMATION

The central difference operator is often used as approximation of the gradient in the volume rendering. For the unit spacing, the central difference operator is:

$$\begin{aligned}&\nabla \mathrm{f}(\mathbf{x}_i) = \nabla \mathrm{f}(x_i \quad y_i \quad z_i) \approx \\ &\{[\mathrm{f}(x_{i+1} \quad y_j \quad z_k) - \mathrm{f}(x_{i-1} \quad y_j \quad z_k)]/2, \\ &[\mathrm{f}(x_i \quad y_{j+1} \quad z_k) - \mathrm{f}(x_i \quad y_{j-1} \quad z_k)]/2, \\ &[\mathrm{f}(x_i \quad y_j \quad z_{k+1}) - \mathrm{f}(x_i \quad y_j \quad z_{k-1})]/2\}\end{aligned} \quad . \tag{8}$$

The derivative $\beta^{d3}(x)$ of the cubic B-spline can be calculated from (5). Derivatives of the B-spline are shown in Figure 4.

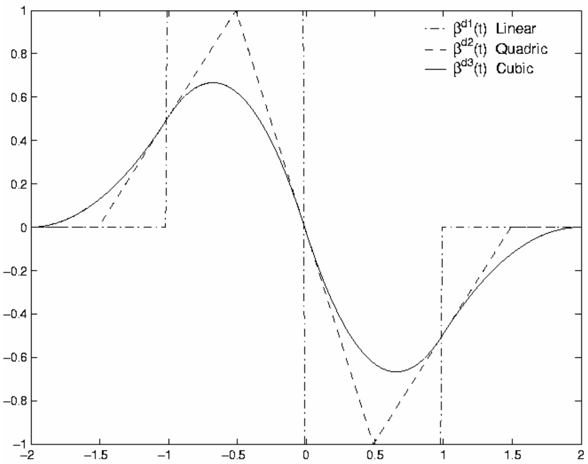

Figure 4. Derivative of the B-splines (degree 1 to 3).

The partial derivative of the three-dimensional filter kernel in direction of parameter t is:

$$\frac{\partial \beta^3(t,u,w)}{\partial t} = \beta^{d3}(t) \cdot \beta^3(u) \cdot \beta^3(w) \tag{9}$$

Two dimensional partial derivative filter kernel is shown in Figure 5.

To present effect of the gradient reconstruction on the appearance of the surface, two approaches are applied. The first one is based on improved central difference operator and the second is exact calculation of the derivative based on the reconstruction kernel. To improve the central difference operator we use combination of three one dimensional $\beta^{d3}(x)$ derivatives along each axe. For the exact calculation of the derivative we use partial derivatives as in equation (9), of the three-dimensional filter kernel for parameters t, u and w.

IV. RESULTS

The three-dimensional test function is proposed by Marschner and Lobb [6]. The size of the volume is 64^3. This test function is very sensitive on reconstruction of the object surface as well as on reconstruction of the derivative. The first row of the Figure 6. shows reconstruction of the surface by the cubic approximation B-spline. The waves are shallow, so this reconstruction is appropriate when smoothing is preferred. The second row shows the reconstruction with cubic least squares B-spline. This reconstruction is appropriate for interpolation of the volumetric data.

To show the influence of the gradient reconstruction two approaches are presented. Improved central difference operator is applied on the left images in Figure 6. for the normal reconstruction and derivative of the cubic B-spline is applied for the derivative reconstruction on the right images. The artifacts due to the gradient reconstruction are obvious.

V. CONCLUSION

Estimation of the normal vector in the volume rendering is very important for the surface perception. Simple approaches such as central difference calculation can introduce alias artifacts in the appearance of the surface smoothness and that could cause incorrect interpretation. We propose for normal estimation calculation of the derivative for the reconstruction cubic B-spline, because it provides smooth normal on the reconstructed surface. Reconstruction with the approximation B-spline is suitable when noise is present in the input data and the least squares B-spline otherwise.

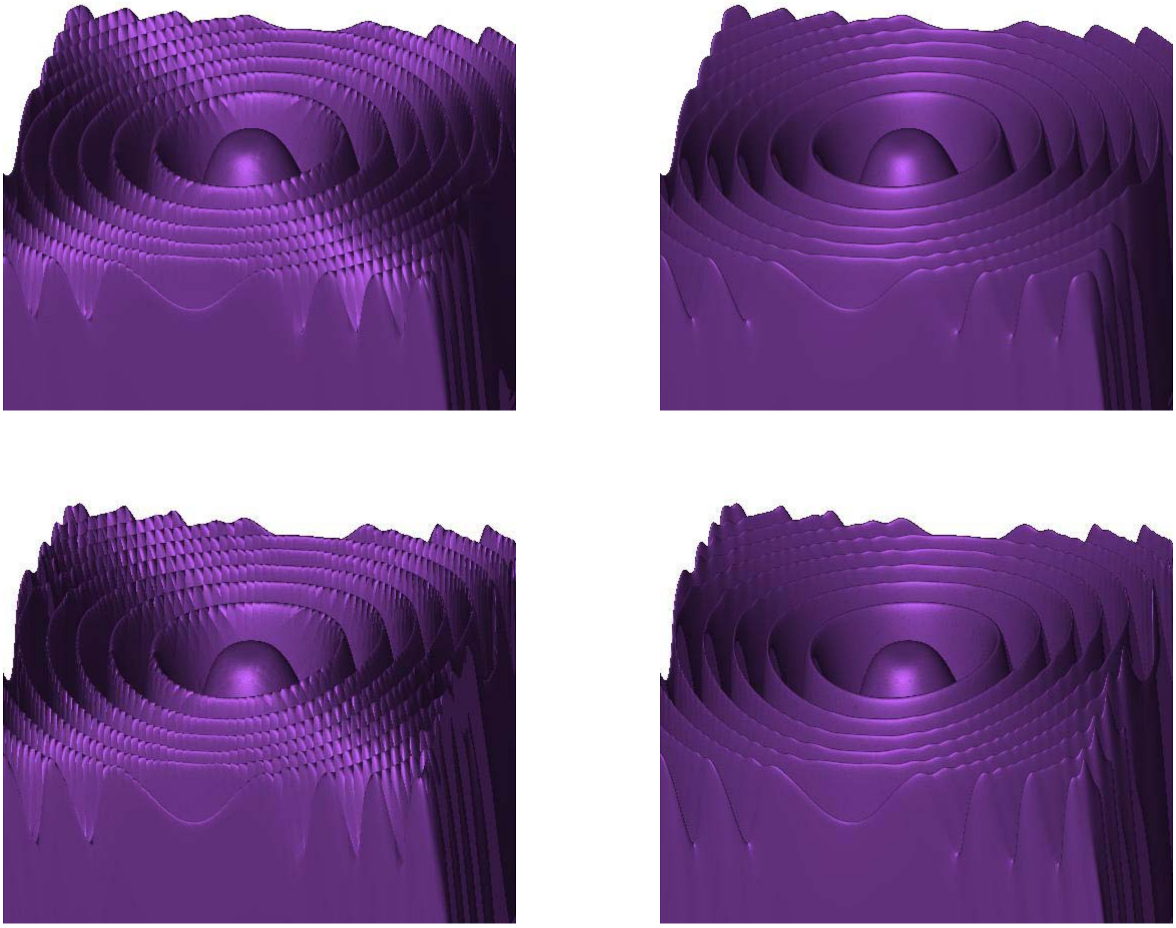

Figure 6. Comparison between gradient estimation with the improved central difference operator and gradient of the B-spline function. First row shows reconstruction with cubic approximation B-spline (waves are shallow), and the second raw shows reconstruction with least squares B-splines (deep waves). Improved central difference is applied for normal estimation (left) and derivative of the cubic B-spline reconstruction kernel (right) are used to show the effect of the derivative reconstruction.

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