Investigation of Fractal Properties in Data Traffic

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Abstract

The goal of this study is to introduce some important topics in today's traffic modeling in high-speed networks. In recent years, a number of studies demonstrated that in wide variety of networking environments the traffic appears to exhibit many unusual characteristics such as heavy-tailed distributions, long-range dependence (LRD) and self-similarity. Since understanding traffic characteristics is very important in network dimensioning and performance prediction, the identification and quantification of these phenomena are in the focus of this paper. Together with the description of the statistical methods used, the analysis results are presented for two data sets taken from real networks.

1 Introduction

The classical models in queuing and network theory based on, for example, the Erlang formulas contain simple assumptions that guarantee the Markovian properties and ensure their analytical tractability. In the early stages of traffic modeling—when the typical case was the telephone traffic—the Poisson process was known as a simple and adequate model of real traffic. Nowadays, with a surprisingly rapid rate of the evolution of communication technology, we know much more about traffic flows of different kinds. Let's take a look at some of them which we are using in our everyday's life: the wide area TCP traffic which provides the Internet connection, the possibility for e-mail; the FTP traffic for file transfers; the TELNET traffic for external accessing; the video conferencing data; etc. Statistical analysis of number of data sets selected from this traffic mix show that some properties cannot be explained by Poisson-like models. Analysis of these data is challenging since there is strong evidence that the classical modeling assumptions (such as independence or the lack of long memory) do not hold any longer.

In recent years, a number of studies, [1], [2], [5], [8], [15], and [19], demonstrated that in certain environments, the traffic appears to exhibit many unusual characteristics such as *heavy-tailed distributions*, *long-range dependence* and *self-similarity*. In some of these publications useful analytical methods—which were used to identify and quantify these properties—can be found with detailed (or less detailed) descriptions.

Since understanding traffic behavior and characteristics is very important to network designers and system analysts in network dimensioning and performance prediction, there are needs to study and understand the heavy-tailed and self-similar properties of today's network traffic. In this paper the mathematical side of these phenomena is addressed. First, approaches and results which researchers had reached in their studies of this area is summarized. Second, our analysis to test the fractal behavior of measured data including heavy tail and long-range dependence tests are presented.

2 Background

Before turning to the main point of this study, this chapter introduces the basic concepts of fractal traffic characteristics.

2.1 Heavy-tailed distributions

The concept heavy-tail can be found in many environments. Heavy-tailed distribution arises in the set of cities have all the people, the set of words have all the use, the set of earthquakes do all the damage, etc. Let's look at an example: consider a variable that represents a waiting time. For waiting time with a light-tailed distribution, the longer we have waited, the sooner we are likely to be done. In contrast, for waiting time with a heavy-tailed distribution, the longer we have waited, the longer is our expected future waiting time [2].

To be more specific, let X be a random variable with distribution function F concentrating on $[0, \infty)$.

Definition 1 *[16] F is said to be heavy-tailed with index α, if*

$$1 - F(x) = x^{-\alpha} L(x), \quad \text{as} \quad x \to \infty, \quad \alpha > 0 \quad (1)$$

where L is slowly varying at ∞, i.e., $\lim_{x\to\infty} L(tx)/L(x) = 1$, $t > 0$.

If $\alpha < 2$, the distribution has infinite variance, and if $\alpha < 1$, it has infinite mean.

For example, the simplest case of heavy-tailed distributions is the so-called Pareto distribution. In this case, $L(x) \equiv 1$, so the distribution function of Pareto is $F(x) = 1 - x^{-\alpha}$. The difference between exponential tails and heavy tails can be seen on Figure 1.

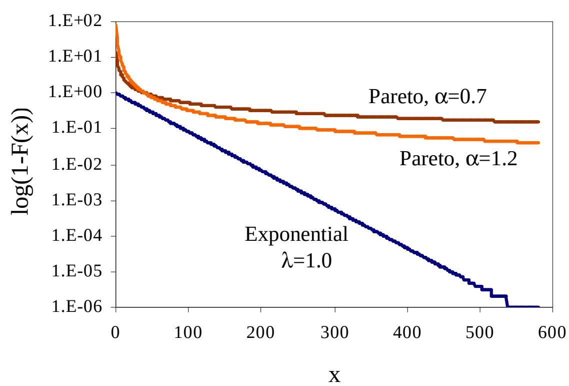

Figure 1: The distribution tails of exponential and Pareto distributions

In communications, heavy-tailed distributions have been used to model number of traffic flows like sizes of Unix files or frame sizes of variable bit rate video.

2.2 Long-range dependence

The autocorrelation function $r(k)$:

$$r(k) = \frac{E\left\{(X_n - E(X))\,(X_{n+k} - E(X))\right\}}{E\left\{(X_n - E(X))^2\right\}}$$

of a long-range dependent stochastic process decays hyperbolically as the lag increases. As a result, $\sum_{k=1}^{n} r(k) \to \infty$ when n grows in infinity. This non-summability of the correlations captures the intuition behind long-range dependence, namely that while high-lag correlations are all individually small, the cumulative effect is of importance and gives rise to features which are drastically different from those of the more conventional, i.e., short-range dependent processes. The latter are characterized by a geometric decay of the autocorrelations, i.e., $r(k) \sim a^k, 0 < a < 1$ as $k \to \infty$, resulting in the summable autocorrelation function $0 < \sum_k r(k) < \infty$. By definition,

Definition 2 *[9] X_t is called a stationary process with long range dependence (LRD or long memory) if there exists a real number $H \in (0.5, 1)$ and a constant $c_r > 0$ such that*

$$\lim_{k\to\infty} \frac{r(k)}{c_r k^{2H-2}} = 1, \quad (2)$$

where H is called the Hurst parameter and measures the degree of LRD.

2.3 Self-similarity

The unifying concept underlying fractals, chaos, and power laws is self-similarity. Self-similarity, or invariance against changes in scale or size, is an attribute of many laws of nature and innumerable phenomena in the world around us [17]. A phenomenon that is self-similar looks the same or behaves the same way when being viewed at different scales on a dimension. The dimension can be considered in space or time. In our study of the traffic data, we concentrate on the time series and stochastic processes that exhibit self-similarity with respect to time.

Figure 2 is a comparison of time series plots of a self-similar and a non-self-similar stochastic process. Note that self-similarity does not mean that the time

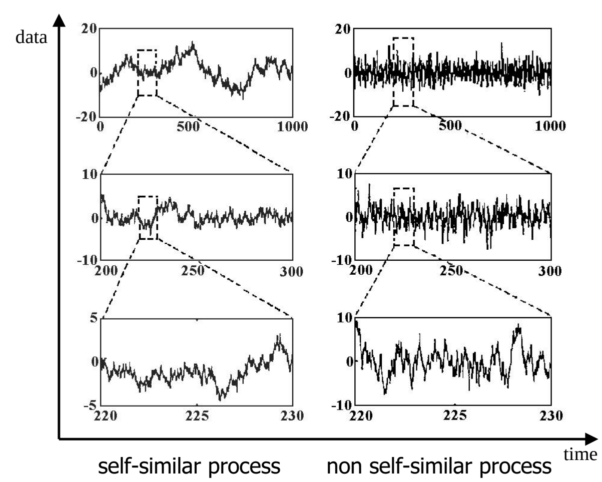

Figure 2: Comparison of self-similar and non self-similar process

function is exactly reproduced at different time scales. Instead we can observe similar burstiness of traffic at different time scales in case of self-similar traffic. This difference is noticeable between the self-similar process (left side) and the non self-similar process (right side).

Definition 3 *[15] Consider the process X, and define the m-aggregated time series $(m = 1, 2, \ldots)$*

$$X^{(m)} = \{X_k^{(m)} : X_k^{(m)} = \frac{1}{m}(X_{mk-m+1} + \ldots + X_{mk}), \quad k = 1, 2, \ldots\} \quad (3)$$

Let $r^{(m)}(k)$ be the autocorrelation function of the aggregated process. The process X is said to be

(a) exactly self-similar, if $X \stackrel{d}{=} m^{1-H} X^{(m)}$, i.e., if X is identical to $X^{(m)}$ within a scale factor in a finite dimensional distribution sense.

(b) exactly second-order self-similar, if $r^{(m)}(k) = r(k)$, $k \geq 0$.

(c) asymptotically second-order self-similar, if $r^{(m)}(k) \approx r(k)$, $k, m \to 0$.

3 Performance implications

The first study of fractal behavior of traffic was published in [5] by researchers at Bellcore. Based on extensive measurements made on a local area Ethernet network they concluded that the traffic possesses self-similar properties and discovered that the higher the load on the Ethernet, the higher the estimated Hurst parameter H of the traffic or, equivalently, the higher the degree of self-similarity. This result is vital because it is precisely at high loads that performance issues become most relevant.

An equally important result of the Ethernet analysis was the inadequacy of traditional queuing models to predict performance. For example, a common assumption concerning data traffic is that multiplexing a large number of independent traffic streams results in a Poisson process. It would be the right assumption, if we disregard some limits in the environment. Study in [6] points out that this assumption and the resulting queuing analysis led Asynchronous Transfer Mode (ATM) switch vendors to produce first generation switches with small buffers ($10-100$ cells). When these switches were deployed in the field and exposed to real traffic, cell losses far beyond those expected were experienced and resulted in a redesign of the switches.

For a queuing system, such as ATM, Frame Relay, 100BASE T, Wide Area Network (WAN) routers, and generally for statistical multiplexers, if the input data of the queue is self-similar, then increased delays and increased buffer size requirements will be experienced [5]. The queuing performance of actual ATM traffic exhibiting self-similar characteristics was investigated in [13]. It has been found that the upper time scale which determines the range of correlations of interest from cell loss point of view is approximately ten times the buffer size. However, this time scale also depends on the load.

For better traffic control, the traffic profile can be changed, for example by traffic shaping. However, the fractal characteristics seem to be rather robust with respect to shaping and can difficultly be removed [11].

The nature of traffic self-similarity may be inherent in the data traffic source, for example the Variable Bite Rate (VBR) video traffic [8], or may be the result of numerous interactions with the network, for example the Transmission Control Protocol-based (TCP) traffic [7]. In the first case, the traffic behavior remains dependent of the network conditions under which it is sent, it can be effectively managed in the context of admission control and resource allocation subject to Quality of Service (QoS) guarantees. In the other case, the traffic self-similarity changes its behavior depending on the congestion status, retransmission scheme (different TCP version), number of concurrent users, request file size (for Web), File Transfer Protocol (FTP) file size, and so on. Some cases, non-stationarity can be detected in measured traffic which can also provide alternative modeling approaches to fractal traffic modeling [12]. In both cases the traffic is difficult to be characterized and modeled. From traffic engineering point of view, it yields to a difficult traffic control.

This summary took part in a wide area and dealt with different kinds of network traffic and based on the results, self-similarity and heavy-tailedness seem to be good structures in high-speed network modeling. Although their application can explain and address many problems in the traffic behavior, it does not mean that these models are the best and the only solution for the modern traffic modeling. The studies on self-similarity and heavy-tailedness is complicated and still the subject of the ongoing research all over the world.

4 Measurements

These measurements are freely available from the Internet Traffic Archive [18].

4.1 IP traffic traces

This trace is the result of an hour long Ethernet measurement ran from 14:00 to 15:00 on Friday, January 21, 1994. The tracing was done on the Ethernet DMZ network over which flow all traffic into or out of the Lawrence Berkeley Laboratory, located in Berkeley, California. The raw traces were made using tcpdump on a Sun SparcStation using the BPF kernel packet filter.

The measurement captured arrival timestamps in microsecond precision of TCP, UDP, TCP SYN/FIN/RST, encapsulated IP and other IP packets in five files, respectively. After processing these files, a set of around $300,000$ IP packet arrivals in consecutive time-windows, equally $0.021 sec$, was selected for analysis.

4.2 WWW traffic traces

These measurements were done at Boston University's Computer Science Department. In order to capture all of the Web activity on a Local Area Network (LAN), researchers modified the Web browser NCSA Mosaic and installed it for general use. After that Mosaic browsers could write down all working activities of browsers in a *log* file. Each line in a *log* corresponds to a single URL requested by the user; it contains the machine name, the timestamp when the request was made, the user id number, the URL, the size of the document (including the overhead of the protocol) and the object retrieval time in seconds (reflecting only actual communication time, and not including the intermediate processing performed by Mosaic in a multi-connection transfer). These traces contain records of the HTTP requests and user behavior of a set of Mosaic clients running in a general computing environment at the department. This environment consists principally of 37 SparcStations 2 workstations connected in a local network, which is divided in two sub-nets. Each workstation has its own local disk; *log*s were written to the local disk and subsequently transferred to a central repository. The data collection then took place in about 5 months from 17 January 1995 until 8 May 1995.

In this study we consider only the characteristics of the file sizes transmitted over the Internet. So a small C routine was implemented to subtract this information from over 6000 *log* files. Around 230, 000 unique file sizes were recorded. As the suggestion of some previous studies, this data set—called the Web file sizes data set or the WFS set—may contain heavy-tailed properties.

5 Analysis and results

5.1 Testing for heavy tails

A heavy-tailed process has its own feature that the tail of the distribution decays much more slowly than exponential. This is the main point of methods used to detect the heavy tail. Moreover, to estimate the scale parameter α (see Definition 1) various exploratory plotting techniques are available. They are based on the Hill estimator and the modified QQ-plot. Another considerable method is the DeHaan's moment estimator. These statistics are shortly described in the followings.

Hill estimator Suppose $X_1, X_2, \ldots, X_n$ are independent, identically distributed (iid.) random samples from a distribution F and $X_{1,n} \geq X_{2,n} \geq \ldots \geq X_{n,n}$ are the order statistics. If F is a heavy-tailed distribution (see Definition 1), The Hill estimation of index α takes the following form [16]:

$$\hat{\alpha} = \hat{\alpha}_{k,n} = \left(\frac{1}{k} \sum_{j=1}^{n} \log X_{j,n} - \log X_{k,n} \right)^{-1} . \quad (4)$$

The Hill estimation of WFS data set can be seen on Figure 3. The plot goes fast to its stable value 0.67. It is the estimate of index α of the WFS distribution tail.

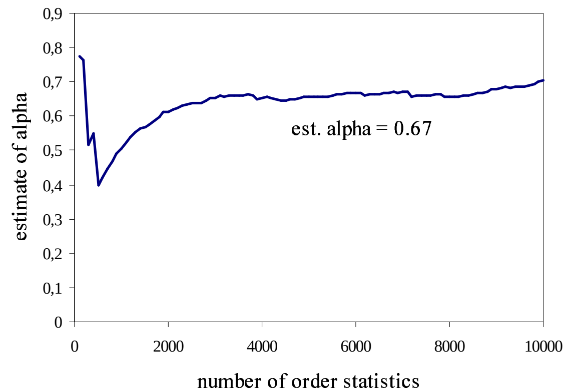

Figure 3: The Hill plot estimation of WFS data set

Modified QQ-plot The main idea of using modified QQ-plot follows the assumption: if $X_1 \geq X_2 \geq \ldots \geq X_k$ are samples from a distribution F and k is large enough, the distribution function F at $x = X_j$ can be estimated by

$$P(x < X_j) = F(X_j) \approx 1 - \frac{j}{k+1}. \quad (5)$$

From this, the modified QQ-plot is defined as follows [10]: Let $X_1 \geq X_2 \geq \ldots \geq X_k = u$ be the order statistics of a distribution, which is approximately Pareto. Then the plot of

$$\left\{ \left(\log X_j - \log u, \ -\log\left(\frac{j}{k+1}\right) \right), \quad 1 \leq j \leq k \right\} \quad (6)$$

should roughly look like a straight line with slope α.

Figure 4 is the modified QQ-plot of WFS data set. It can be seen on the figure that the plot is not exactly a straight line but a regression line can be fitted over points with small deviations. The slope provides the estimate of α to be 0.73.

DeHaan's moment estimator According to [16], DeHaan's moment estimator is defined as follows: Let $X_1 \geq X_2 \geq \ldots \geq X_n$ be the order statistics from a random sample of size n. Define for $r = 1, 2$ and for k upper-order statistics

$$H^r_{k,n} = \frac{1}{k} \sum_{i=1}^{k} \left(\log \frac{X_i}{X_{k+1}} \right)^r . \quad (7)$$

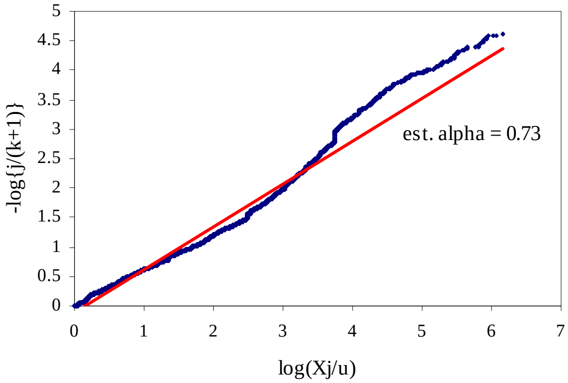

Figure 4: The modified QQ-plot estimation of WFS data set

DeHaan's estimate of index α can be calculated by the form

$$\hat{\alpha} = \hat{\gamma}^{-1} = \left(H^1_{k,n} + 1 - \frac{1}{2\left(1 - \frac{(H^1_{k,n})^2}{H^2_{k,n}}\right)} \right)^{-1}. \quad (8)$$

Figure 5 shows the plot result generated by DeHaan's testing methods. The estimate of α in this case, 0.65, is a bit smaller than in the Hill's case. It may be the effect of smoothing technique used in DeHaan's algorithm.

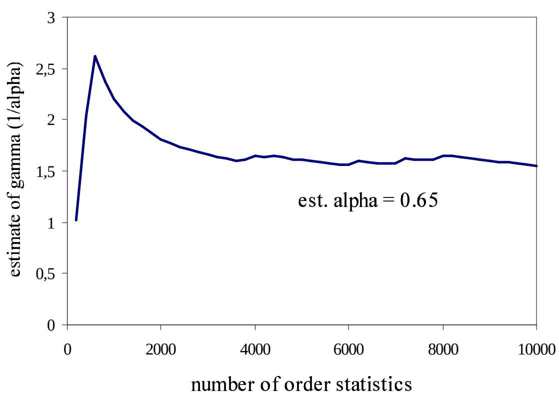

Figure 5: DeHaan's estimate of index α of WFS data set

We can conclude that the set of file sizes transferred over the Internet seems to fit well the Pareto distribution with index α about 0.7. Similar results were also explored by other researchers in [1]. As discussed in this paper, it may be an evidence of self-similar WWW traffic.

5.2 Testing for LRD and self-similarity

For estimating the Hurst parameter, a number of algorithms has been worked out. Algorithms were described, for example, in [3], [4], and [16]. In this section four widely used methods: variance-time plot, R/S plot, periodogram, and Whittle estimator are summarized. However, by using LRD tests and other statistical tests, it is difficult to make reliable conclusions about the self-similarity of traffic. Note that in most cases statistical methods can not prove an empirical data set to be produced by an exactly self-similar process. Instead, as shown in Definition 3, a data set may only have the property of second-order or asymptotically second-order self-similarity.

Variance-time plot For a stationary process with LRD, the following property can be proven:

$$\mathrm{var}(X^{(m)}) = \frac{1}{m^{2-2H}} \mathrm{var}(X), \quad (9)$$

so

$$\log \mathrm{var}(X^{(m)}) = \log \mathrm{var}(X) + (2H - 2) \log m. \quad (10)$$

Because $\log \mathrm{var}(X)$ is a constant independent of m, if we plot $\mathrm{var}(X^{(m)})$ versus m on a log-log graph, the result should be a straight line with a slope of $(2H - 2)$. The plot can be easily generated from the data series X by generating the aggregated processes of X at different levels of m and then computing the empirical variance. Plot with slope values between -1 and 0 suggests LRD.

The variance-time plot of IP data set is drawn on Figure 6. It is surprising that there is a breaking point on the plot. From a certain large value of time unit, the slope takes up a bigger value. Anyway, by the Definition 2 of LRD it is an asymptotic characteristics, so the Hurst parameter should be estimated by the slope of the higher aggregation levels. The estimate of H was 0.83.

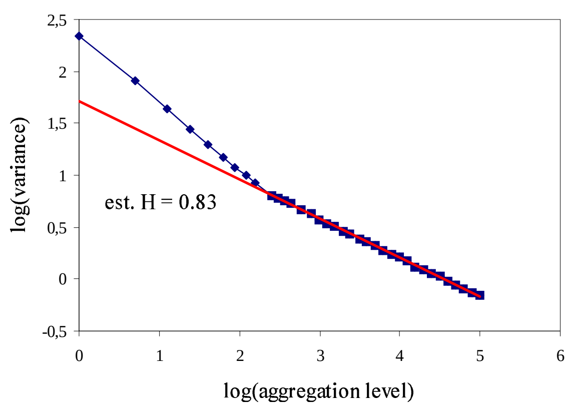

Figure 6: Variance-time plot of IP data set

R/S plot For a stochastic process X defined in discrete time $\{X_j : j = 1, 2, \ldots, n\}$, the rescaled adjusted range or R/S-statistics of X over a time interval n is defined as the ratio R/S with:

$$\begin{aligned} R &= \max\{W_i : i = 1, 2, \ldots, n\} \\ &\quad - \min\{W_i : i = 1, 2, \ldots, n\}, \\ S &= \sqrt{\mathrm{var}(X)} \end{aligned} \tag{11}$$

where $W_i = \sum_{k=1}^{i}(X_k - \bar{X}), i = 1, 2, \ldots, n$ and $\bar{X} = (1/n)\sum_{i=1}^{n} X_i$. It can be proven for any stationary process with LRD that the ratio R/S has the following characteristics for large n:

$$\frac{R}{S} \approx \left(\frac{n}{2}\right)^H \tag{12}$$

which is known under the name *Hurst effect*. Thus if we plot R/S versus n on a log-log graph $\log(R/S) \approx H \log n - H \log 2$, the plot should fit a straight line with slope H.

Using this algorithm, the R/S analysis of IP data set was provided and can be seen on Figure 7. Data points are scattered around a straight line, which means that IP packet arrivals seem to be LRD with Hurst parameter $H = 0.84$, which is the estimate from the slope of regression line.

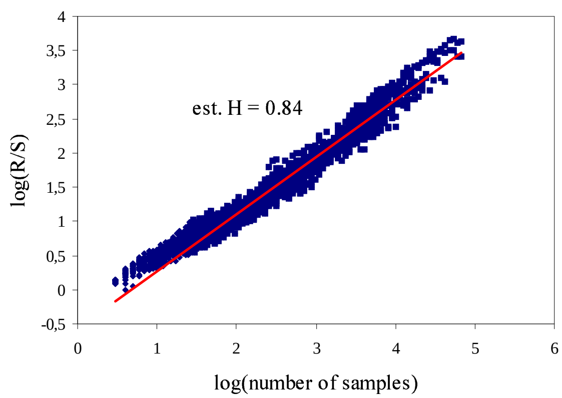

Figure 7: R/S analysis of IP data set

Periodogram Another alternative way to estimate the Hurst parameter of LRD is the periodogram plot. The power spectral density of a LRD process obeys a power law near the origin

$$\lim_{\nu \to 0} \frac{f(\nu)}{c_f |\nu|^{1-2H}} = 1. \tag{13}$$

where c_f is a constant, ν is the frequency, and $f(\nu)$ is the power spectral density function, that is the Fourier transform of the autocorrelation function $r(k)$. Periodogram provides a fast estimate of $f(\nu)$:

$$I(\nu) = \frac{1}{2\pi n} \left| \sum_{k=1}^{n} (X_k - \bar{X}) e^{ik\nu} \right|^2. \tag{14}$$

So if the periodogram is plotted against small values of frequency on a log-log graph, the plot should be a straight line with slope $(1 - 2H)$.

The periodogram plot of IP data set is shown on Figure 8. The estimate of H in this case is 0.82.

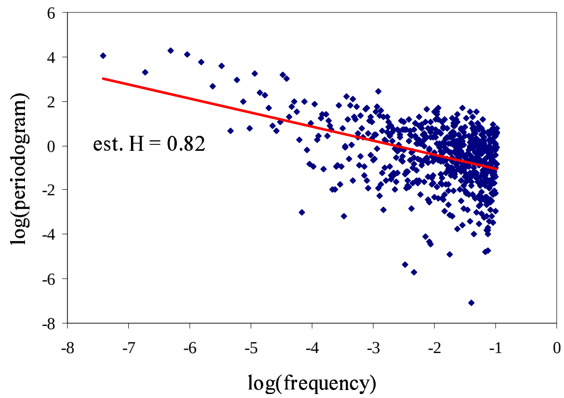

Figure 8: Periodogram plot of IP data set

Whittle estimator Whittle estimator is a concrete application of maximum likelihood method (MLE). On the other hand, the Whittle estimation is based on the periodogram. So in most cases these methods provided the same estimates of the Hurst parameter.

The Whittle estimator was suggested to estimate the Hurst parameter of Fractional Gaussian Noise (FGN), which is an exactly self-similar process. If data is from a FGN process, the estimate of H is the value that minimizes the function $Q(H)$:

$$Q(H) = \int_{-\pi}^{\pi} \frac{I(\nu)}{f(\nu, H)}\, \mathrm{d}x + \int_{-\pi}^{\pi} \log f(\nu, H)\, \mathrm{d}\nu. \tag{15}$$

To calculate the value of $Q(H)$, we should consider the exact behavior of the spectral density $f(\nu)$ of the process close to the origin. The Whittle estimator is more robust testing method than the others, and it also provides the confidence interval (95%) of the calculated Hurst value.

Figure 9 shows the Whittle estimation of IP data set. The result is 0.83 with confidence interval (0.81, 0.85).

So, by going through four testing methods, variance-time plot, R/S plot, periodogram, and Whittle estimator, the IP packet arrival process seems to exhibit LRD with $H \approx 0.83$. (Although Whittle estimator provides the good estimate of Hurst parameter, it is a bit soon to make a conclusion about self-similarity of this process.)

Note that IP packet arrivals and WWW file sizes are not the only samples of traffic flows which are analyzed to provide fractal properties in recent sev-

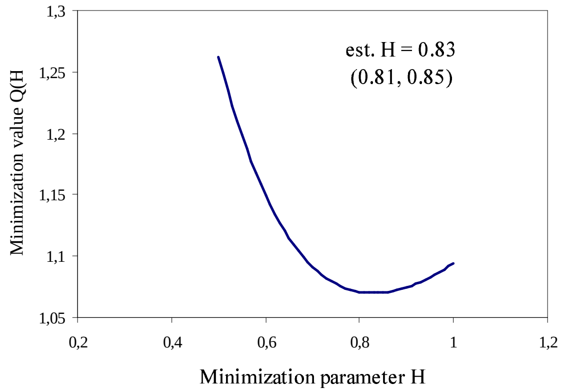

Figure 9: The Whittle estimate of H of IP data set

eral years. This topic is still an open area of traffic modeling for more studies and researches.

6 Conclusion

In this paper, after summarizing the results and observations of researchers about self-similar and heavy-tailed properties of some high speed network traffic types in recent years, the brief mathematical background of them was discussed. These properties of high-speed networks have a strong impact on the performance of the networks. Then the analytical methods testing for self-similarity and heavy-tailedness were described. It included the algorithms of useful statistical methods: variance-time plot, R/S plot, periodogram, and Whittle estimator for LRD and modified QQ-plot, DeHaan's moment method and Hill estimator for heavy-tailedness. Using these methods, two data sets from real traffic measurements were analyzed.

Testing with the IP packet arrivals data set provided LRD property with asymptotic characteristics. The estimate of H in various estimating methods was about the same, 0.83. The confidence interval (95%) given by the Whittle estimator was $(0.81, 0.85)$. The distribution of file sizes transferred on the Internet from a given threshold may be well modeled by heavy-tailed (Pareto) distributions. The estimates of scale parameter α are about the same and equals 0.7. Heavy-tailed file sizes may be a cause of self-similar WWW traffic which was discussed in [1].

More recent researches indicate that complex traffic (ATM or Internet) is consistent to an even more complex structure compound to self-similarity. This research [14] suggests that traffic has multifractal nature. The application of multifractals for traffic modeling is a hot research topic.

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