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Criteria to Design Chaotic Self-Similar Traffic Generators

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SUMMARY A self-similar behavior characterizes the traffic in many real-world communication networks. This traffic is traditionally modeled as an ON/OFF discrete-time second-order self-similar random process. The self-similar processes are identified by means of a polynomially decaying trend of the autocovariance function. In this work we concentrate on two criteria to build a chaotic system able to generate self-similar trajectories. The first criterion relates self-similarity with the polynomially decaying trend of the autocovariance function. The second one relates self-similarity with the *heavy*-tailedness of the distributions of the sojourn times in the ON and/or OFF states. A family of discrete-time chaotic systems is then devised among the countable piecewise affine Pseudo-Markov maps. These maps can be constructed so that the quantization of their trajectories emulates traffic processes with different Hurst parameters and average load. Some simulations are reported showing how, according to the theory, the map design is able to fit those specifications.
key words: *self-similar traffic, chaotic systems, heavy-tailed distribution, piecewise-affine Markov maps*

1. Introduction

The traffic on Ethernet LANs shows the presence of burstiness across a wide range of time scales, as reported in [1]–[3], where measures in different time instants relative to single and multiple data sources, intranet and internet scenarios have been performed. This "fractal-like" behavior, is usually referred to as *self-similarity* [4]–[6].

The conventional Markov or Poisson processes cannot model this burstiness in a satisfactory way [7] since they give raise to the classical behavior that becomes smoother (less bursty) as the observation time scale increases or as the number of traffic source increases. On the contrary, second-order self-similar processes are nowadays one of the most credited model for such bursty traffic.

The properties of a self-similar process, can be explained by referring to the autocovariance function of the aggregated process. The aggregated process of order m is obtained from the original one by averaging the original samples on non-overlapping blocks of size m. In particular: let τ be the elementary observation interval; let $x_i = x(i\tau)$ be the i-th sample of the stationary process X, such that $x_i \in \{0,1\}$; so, the i-th sample $x_i^{(m)}$ of the aggregated process $X^{(m)}$ is defined as $x_i^{(m)} = (1/m)\sum_{j=0}^{m-1} x_{mi+j}$, where $x_i^{(1)} = x_i$ and m is called aggregation factor. The autocovariance function, $C^{(m)}(k)$, of the aggregate process is defined as $C^{(m)}(k) = E[x_i^{(m)} x_{i+k}^{(m)}] - E^2[x_i^{(m)}]$, and it does not depends on i in virtue of the process stationarity.

We assume that a process is asymptotically second-order self-similar if a $H \in]0.5, 1[$, an $\beta > 0$ and an $m' > 0$ exist such that if $m \gg m'$ and k is large enough:
1) $\frac{C^{(m)}(0)}{C^{(m')}(0)} \sim \left(\frac{m}{m'}\right)^{2H-2}$
2) $\frac{C^{(m)}(k)}{C^{(m)}(0)} \sim \frac{C^{(m')}(k)}{C^{(m')}(0)} \sim \beta k^{2H-2}$
where H, usually called Hurst parameter, gives the degree of the source burstiness. The closer to 1 the H the more self-similar the process. If H is close to 0.5 the process shows a Poisson behavior [1]. The other parameter characterizing a self-similar traffic source is the activity index, $P_{ON} = E[x_i]$, which represents the activity time fraction. Let us observe that the traffic source activity can be view as a succession of ON and OFF states.

Many studies [8]–[11] are focusing the attention on the generation of self-similar traffic, in order to simulate the performance of network systems, like queuing systems, in different traffic conditions.

In this work we report a general framework identifying two possible criteria to build a system showing self-similar behavior. The first criterion gives a general condition directly derived by the polynomial trend of the autocovariance function of the aggregated process. The second one proves that by selecting a chaotic system with ON or/and OFF time distributions *heavy*-tailed, the first criterion is implicitly satisfied and then the trajectories show self-similar behavior.

Let us recall that a distribution is *heavy*-tailed if the complementary distribution is of the form $P(K > k) \approx k^{-\Xi}L(k)$, $0 < \Xi < 2$, where $L(k)$ is a slowly varying function at infinity [8], [13]; while it is *light*-tailed if is $P(K > k) \approx \gamma^{-k}L(k)$, $\gamma > 0$ [12], [13].

The traffic generator is based on a family of discrete-time chaotic system which among the count-

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able piecewise-affine Markov maps. The parameters of these maps are optimized by means of a Lagrange-based iterative procedure to allow reproduction of asymptotically arbitrary ON/OFF time distributions as well as arbitrary P_{ON}. The resulting maps have been tested to evaluate the theory validity.

The paper is structured as follows: in Sect. 2 the general mathematical model is explained, in Sects. 3 and 4 the first and second self-similarity criteria are reported and in Sect. 5 the Lagrange-based optimization technique is described along with the generator design procedure. In Sect. 6 such a procedure is extensively validated by means of numerical simulations. Some conclusions are finally drawn in Sect. 7.

2. Mathematical Model

Let us consider a one-dimensional discrete-time nonlinear dynamical system, $f\colon [0,1] \to [0,1]$, such that the iteration $y_{i+1} = f(y_i)$ identifies the system trajectory. Let f^i identify the i-th iterate of the function f. The traffic source is modeled by considering a succession of activity (ON) and non-activity (OFF) states, represented by the previously defined random variable x_i. This is obtained by using a trivial binary quantization function $Q\colon [0,1] \to \{0,1\}$, such that $x_i = Q(y_i) = 1$ if $y_i < d$, with $d = 1/2$, and $x_i = Q(y_i) = 0$ otherwise.

The statistical properties of the resulting process depend on the statistical features of the trajectories of the dynamical system f. Hence, we resort to the well-known mathematical tools of the statistical theory of dynamical systems [14] and note that if the initial condition y_0 is a random variable distributed according to the probability density $\rho_0\colon [0,1] \mapsto \mathbb{R}^+$ then the subsequent points y_i are distributed according to the probability densities ρ_i that, if f is at least *mixing* [15], converge to a limit probability density $\bar{\rho}$ which is the density of the invariant measure of f.

With this we may go back to the computation of the autocovariance of the $x_i^{(m)}$ process and note that the expectation in the autocovariance is taken with respect to all the possible process outcomings, then both the starting point, y_0, and the subsequent evolution have to be considered:

$$C^{(m)}(k) = \lim_{W\to\infty} \frac{1}{W} \sum_{i=0}^{W-1} \int_0^1 \left(\frac{1}{m} \sum_{n=1}^{m} Q(f^{mi+n}(y_0)) \right) \left(\frac{1}{m} \sum_{j=1}^{m} Q(f^{mi+j+mk}(y_0)) \right) \rho_0(y_0)dy_0 - P_{ON}^2$$

which can be re-arranged into:

$$\begin{aligned} &C^{(m)}(k) \\ &\quad = \lim_{W\to\infty} \frac{1}{Wm^2} \sum_{i=0}^{W-1} \sum_{n=1}^{m} \sum_{j=1}^{m} \int_0^1 Q(f^n(y_{mi})) \\ &\qquad \cdot Q(f^{j+mk}(y_{mi}))\rho_{mi}(y_{mi})dy_{mi} - P_{ON}^2 \end{aligned}$$

Then, by observing that $\rho_{mi} \approx \bar{\rho}$, for m large enough, we have:

$$\begin{aligned} &C^{(m)}(k) \\ &\quad = \frac{1}{m^2} \sum_{n=1}^{m} \sum_{j=1}^{m} \int_0^1 Q(f^n(y))Q(f^{j+mk}(y)) \\ &\qquad \cdot \bar{\rho}(y)dy - P_{ON}^2 \\ &\quad = \frac{1}{m^2} \sum_{n=1}^{m} \sum_{j=1}^{m} \int_0^1 Q(y)Q(f^{|mk+j-n|}(y)) \\ &\qquad \cdot \bar{\rho}(y)dy - P_{ON}^2 \end{aligned} \tag{1}$$

where a further shift of either n or $mk + j$ time steps is considered and compounded in the absolute value.

The activity index P_{ON} can be computed directly using the Birkoff ergodic theorem [14]:

$$\begin{aligned} P_{ON} &= \lim_{W\to\infty} \frac{1}{W} \sum_{n=0}^{W-1} \int_0^1 Q(f^n(y))\rho_0(y)dy \\ &= \int_0^1 Q(y)\bar{\rho}(y)dy = \int_d^1 \bar{\rho}(y)dy \end{aligned} \tag{2}$$

where d is the quantization threshold.

3. First Self-Similarity Criterion

Once that a suitable form for the average traffic and the autocovariance function are derived, a necessary condition for self-similarity generation can be obtained.

Theorem 1: The process $x = Q(y)$ is asymptotically self-similar if

$$\begin{aligned} C(k) &= C^{(1)}(k) \\ &= \int Q(y)Q(f^k(y))\bar{\rho}(y)dy - P_{ON}^2 \sim k^{2H-2} \end{aligned} \tag{3}$$

Proof: By assuming m and k large enough, using (1) and (3), so that $mk + j - n > 0$, we have:

$$\begin{aligned} &C^{(m)}(k) \\ &\quad \sim \frac{1}{m^2} \sum_{n=1}^{m} \sum_{j=1}^{m} (mk+j-n)^{2H-2} \\ &\quad \sim \frac{1}{m^2} \int_1^m \int_1^m (mk+j-n)^{2H-2} dj dn \\ &\quad \sim \frac{-m^{2H-2}}{(1-2H)2H} \left[(k-1)^{2H} + (k+1)^{2H} - 2k^{2H} \right] \\ &\quad \sim m^{2H-2} k^{2H-2} \end{aligned}$$

where the limit of the double sum has been found to be asymptotically equivalent to an integral exploiting Riemann upper and lower sums and where the trivial asymptotic equivalence $2H(2H-1)k^{2H-2} \sim (k-1)^{2H} + (k+1)^{2H} - 2k^{2H}$ has been used.

Let us now compute $C^{(m)}(0)$ separating those summands in the nested sums (1) for which $j = n$. Actually their contribution is of the order of P_{ON}/n and is therefore asymptotically negligible when compared to what we expect to find for the whole $C^{(m)}(0)$. Hence, for m large enough (1) can be rewritten as:

$$
\begin{aligned}
&C^{(m)}(0)\\
&\quad\sim \frac{2}{m^2}\sum_{n=1}^{m-1}\sum_{j=n+1}^{m}\int_0^1 Q(y)Q(f^{j-n}(y))\bar{\rho}(y)dy - P_{ON}^2\\
&\quad= \frac{2}{m^2}\sum_{n=1}^{m-1}(m-n)\int_0^1 Q(y)Q(f^{n}(y))\bar{\rho}(y)dy - P_{ON}^2\\
&\quad\sim \frac{2}{m^2}\sum_{n=1}^{m-1}(m-n)n^{2H-2}\\
&\quad\sim \frac{2}{m^2}\int_1^m (m-n)n^{2H-2}dn \approx \frac{1}{(2H-1)H}m^{2H-2}
\end{aligned}
$$

By collecting these two results the Theorem 1 holds with a constant $A = (2H-1)H$. □

Let us underline that Theorem 1 states that the autocovariance function of the aggregate process directly depends on the autocovariance function of the original process.

4. Second Self-Similarity Criterion

Another criterion to create a self-similar chaotic generator is to consider a quantized system with the sojourn distributions in the ON state, or in the OFF state or in both *heavy*-tailed.

We consider a class of maps such that the state space $X = [0,1]$ is partitioned into 2 disjoint intervals $S_1 = [0,1/2[$ and $S_2 = [1/2,1]$. Each of these intervals is further partitioned into a (possibly) countable number of subintervals X_{ij} such that $S_i = \bigcup_j X_{ij}$.

We will additionally assume that the map M is a Piecewise Affine Markov map with respect to the countable set of intervals X_{ij}, i.e. that for any $X_{i'j'}$ and $X_{i''j''}$:

$$
M(X_{i'j'}) \cap X_{i''j''} = \begin{cases} \emptyset \\ X_{i''j''} \end{cases}
$$

and that M is invertible in each X_{ij} and affine in each $X_{i'j'} \cap M^{-1}(X_{i''j''}) \neq \emptyset$. Note that this is a slightly relaxed assumption with respect to the usual requirements that M is affine in each X_{ij}. We assume that for both S_i two subsets can be distinguished: the *returning* subset $\overset{\circlearrowleft}{S}_i$ and the *exiting* subset $\vec{S}_i$. These two subsets are such that $M(\overset{\circlearrowleft}{S}_i) \subseteq S_i$, $M(\vec{S}_i) \cap S_i = \emptyset$, $\overset{\circlearrowleft}{S}_i \cup \vec{S}_i = S_i$, and are union of certain intervals X_{ij} which are labeled accordingly so that $\overset{\circlearrowleft}{S}_i = \bigcup_j \overset{\circlearrowleft}{X}_{ij}$ and $\vec{S}_i = \bigcup_j \vec{X}_{ij}$.

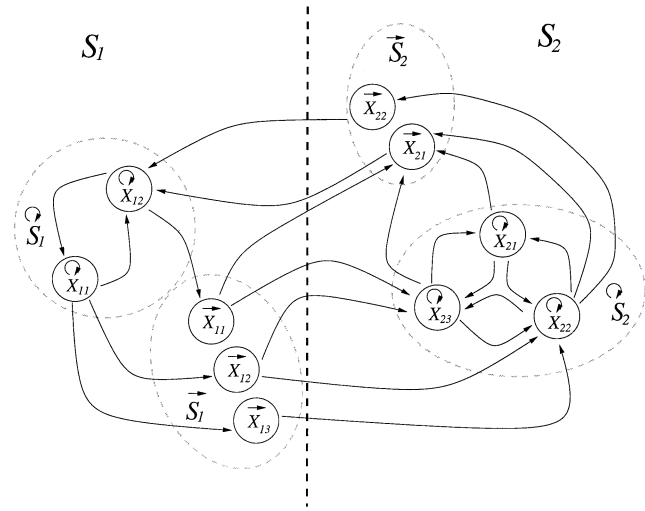

Fig. 1 Macro state schematization of the evolution of f.

Within this constraints the evolution of the system is analogous to that of a countable state system (one for each X_{ij}). Figure 1 shows a possible macro-state schematization of such a system in which the number of Markov intervals X_{ij} is not only countable but also finite.

In general, for a infinite number of X_{ij} the evolution of the system is analogous to that of a Markov chain only when we observe transitions between Markov intervals. On the contrary, transitions between the two macro states S_1 and S_2 can be regulated by extremely complex laws depending on the countable Markov chains in $\overset{\circlearrowleft}{S}_i$ and $\vec{S}_i$.

To exploit this fact we may first define the matrix $\mathcal{K}$:

$$
\mathcal{K}_{i'i''}(k) = \frac{\bar{\mu}(S_{i'} \cap \bigcap_{j=1}^{k} f^{-j}(S_{i''}))}{\bar{\mu}(\vec{S}_{i'})}
$$

where $\bar{\mu}(S_i)$ is the measure of the interval S_i. The $i'i''$-th entry of this matrix contains the probability of moving from $S_{i'}$ to $S_{i''}$ and staying at least k time steps in $S_{i''}$. For $k = 1$ this joint probability can be obviously rewritten as

$$
\mathcal{K}_{i'i''}(1) = \frac{\bar{\mu}(S_{i'} \cap f^{-1}(S_{i''}))}{\bar{\mu}(S_{i'})} \Big/ \frac{\bar{\mu}(\vec{S}_{i'})}{\bar{\mu}(S_{i'})}
$$

where we have highlighted at the numerator the classical definition of the kneading matrix. The matrix $\mathcal{K}(1)$ actually collapses into a normal kneading matrix when the states S_i are made of just one Markov interval X_{ij}. In fact, in this case, our assumptions will prevent the system from assuming the same state for more than 1 time step, hence forcing $S_i = \overrightarrow{S}_i$.

To express the correlation we are aiming at it is now convenient to define two other quantities, namely

$$\mathcal{L}_{i'i''}(k) = \mathcal{K}_{i'i''}(k) - \mathcal{K}_{i'i''}(k+1) \tag{4}$$

$$\begin{aligned}\mathbf{j}_{i'}(k) &= \sum_i \sum_{j=k+1}^{\infty} \bar{\mu}(\overrightarrow{S}_i)\mathcal{L}_{ii'}(j) \\ &= \sum_i \bar{\mu}(\overrightarrow{S}_i)\mathcal{K}_{ii'}(k+1)\end{aligned} \tag{5}$$

where $\mathbf{j}_{i'}(k)$ is nothing but the probability of staying at least $k+1$ time steps in $S_{i'}$ and then change to any other state so that $\mathbf{j}_{i'}(0) = \bar{\mu}(\overrightarrow{S}_{i'})$ and $\mathcal{L}_{i'i''}$ is the probability of moving from $S_{i'}$ to $S_{i''}$ and staying in the latter exactly k time steps.

Finally we may define the matrix $\mathcal{H}(k)$ to be such that $\mathcal{H}_{i'i''}(k)$ is the probability of observing the system in the macro state $S_{i'}$ at a certain time step and observing the system in $S_{i''}$ k time steps after that. It can be easily accepted that $\mathcal{H}(k)$ contains all the information we need to compute $C(k)$.

As a general remark note that, the system remains in the macro-state in which we have observed it for a certain amount of time. Then, it performs a certain number of transition sojourning in each of the intermediate macro state. Finally, it lands in the macro-state in which we observe it at the end of the time lag remaining there at least up to the observation instant.

To write an expression for $\mathcal{H}(k)$ we may formalize this remark considering that in the k time steps between the two observations the system may exhibit $0, 1, 2, \ldots$ macro-state transitions and writing

$$\begin{aligned}\mathcal{H}_{i'i''}(k) &= \sum_{l=k}^{\infty} \mathbf{j}_{i'}(l)\mathcal{I}_{i'i''} \\ &+ \sum_{\substack{l_1 \geq 0 \\ l_2 > 0 \\ l_1+l_2=k}} \mathbf{j}_{i'}(l_1)\mathcal{K}_{i'i''}(l_2) \\ &+ \sum_{\substack{l_1 \geq 0 \\ l_2, l_3 > 0 \\ l_1+l_2+l_3=k}} \sum_{i_1} \mathbf{j}_{i'}(l_1)\mathcal{L}_{i'i_1}(l_2)\mathcal{K}_{i_1i''}(l_3) \\ &+ \sum_{\substack{l_1 \geq 0 \\ l_2, l_3, l_4 > 0 \\ l_1+l_2+l_3+l_4=k}} \sum_{i_1i_2} \mathbf{j}_{i'}(l_1)\mathcal{L}_{i'i_1}(l_2)\mathcal{L}_{i_1i_2}(l_3)\mathcal{K}_{i_2i''}(l_4) \\ &+ \ldots\end{aligned} \tag{6}$$

where $\mathcal{I}$ is the identity matrix and the vector term $\sum_{l=k}^{\infty} \mathbf{j}_{i'}(l)$ accounts for the probability of beings observed in the same state after k time steps when no state transition happens, the following lines accounts for 1 state transition, the following for 2 state transitions, and so on.

We may now define an inner product between vector and matrixes as in:

$$\{\mathcal{A} * \mathcal{B}\}_{i'i''}(k) = \sum_{i_1} \sum_{j=-\infty}^{\infty} \mathcal{A}_{i'i_1}(j)\mathcal{B}_{i_1i''}(k-j)$$

where the usual product between scalar has been replaced by sequence convolution. For a square matrix function we also define $\mathcal{A}^{*p}(k) = \mathcal{A}(k) * \mathcal{A}(k) * \cdots * \mathcal{A}(k)$ p times, and $\mathcal{A}^{*0}_{i'i''}(k) = 1$ if $i' = i''$ and $k = 1$, and zero otherwise.

With these definitions, the expression of $\mathcal{H}$ can be easily rewritten as:

$$\begin{aligned}\mathcal{H}(k) &= \mathrm{diag}\left(\sum_{l=k}^{\infty} \mathbf{j}(l)\right) \\ &\quad + \left\{\mathrm{diag}\, \mathbf{j} * \left[\sum_{j=1}^{\infty} \mathcal{L}^{*(j-1)}\right] * \mathcal{K}\right\}(k)\end{aligned} \tag{7}$$

where the diag$(\cdot)$ function generates a diagonal matrix whose diagonal coincides with the argument vector. Note also that we used convolution instead of finite sums assuming that all the matrix functions vanish for all negative arguments and that $\mathcal{L}_{i'i''}(0) = \mathcal{K}_{i'i''}(0) = 0$ to take into account the $l_j > 0$ conditions of (6).

We further assume that all the matrix functions we defined are summable so that the generic z-transform:

$$\tilde{\mathcal{A}}(z) = \sum_{j=-\infty}^{\infty} \mathcal{A}(j)z^{-j}$$

converges for $|z| > 1$. With this we may now write the z-transform of $\mathcal{H}(k)$ as

$$\begin{aligned}\tilde{\mathcal{H}}(z) &= \frac{\mathrm{diag}[\tilde{\mathbf{j}}(z) - z\tilde{\mathbf{j}}(1)]}{1-z} \\ &\quad + \mathrm{diag}\, \tilde{\mathbf{j}}(z)\left[\sum_{j=0}^{\infty} \tilde{\mathcal{L}}^j(z)\right]\tilde{\mathcal{K}}(z)\end{aligned}$$

and hence

$$\begin{aligned}\tilde{\mathcal{H}}(z) &= \frac{\mathrm{diag}[\tilde{\mathbf{j}}(z) - z\tilde{\mathbf{j}}(1)]}{1-z} \\ &\quad + \mathrm{diag}\, \tilde{\mathbf{j}}(z)\left[\mathcal{I} - \tilde{\mathcal{L}}(z)\right]^{-1}\tilde{\mathcal{K}}(z)\end{aligned} \tag{8}$$

Note now that from (4) and (5) we get

$$\tilde{\mathcal{K}}(z) = \frac{\tilde{\mathcal{L}}(z) - \tilde{\mathcal{L}}(1)}{1-z}$$

$$\tilde{\mathbf{j}}(z) = z\mathbf{j}(0)\frac{\tilde{\mathcal{L}}(z) - \tilde{\mathcal{L}}(1)}{1-z}$$

so that

$$\tilde{\mathcal{H}}(z) = \mathrm{diag}\left[-\frac{z}{1-z}\tilde{\mathbf{j}}(1) + \frac{z\mathbf{j}(0)}{(1-z)^2}(\tilde{\mathcal{L}}(z) - \tilde{\mathcal{L}}(1))\right] \times \left[\mathcal{I} + (\mathcal{I} - \tilde{\mathcal{L}}(z))^{-1}(\tilde{\mathcal{L}}(z) - \tilde{\mathcal{L}}(1))\right]$$

To proceed further we assume that f is completely defined by the two sequences of points

$$a_{1j} = \frac{1}{2}\sum_{l=j+1}^{\infty} \Delta_1(l)$$

$$a_{2j} = 1 - \frac{1}{2}\sum_{l=j+1}^{\infty} \Delta_2(l)$$

depending only on the two probability functions Δ_1 and Δ_2 whose significance will be soon clarified.

We also set $X_{1j} = [a_{1j+1}, a_{1j}]$, $X_{2j} = [a_{2j}, a_{2j+1}]$ and say that f affine in each X_{ij} and such that $f(X_{ij}) = X_{ij-1}$ for $j > 1$ and $f(X_{10}) = S_2$ while $f(X_{20}) = S_1$.

From the Markov property of f with respect to the X_{ij} we have that the invariant density is uniform in each of those intervals.

From this, from the map construction, from the fact that $\overrightarrow{S}_i = X_{i0}$ and from the piecewise-affinity of f we also get that, after a state transition, the state is uniformly distributed in the new S_i. Note now that as long as $x \in \overset{\circlearrowleft}{S}_i$ it passes from X_{ij} to X_{ij-1} at each time step until it reaches $\overrightarrow{S}_i$. Hence, the probability of staying exactly k time steps in S_i is equal to the probability of landing in X_{ik-1} after the state transition. Yet, by construction, such a probability is nothing but $(a_{ik-1} - a_{ik})/(a_{i0} - a_{i\infty}) = \Delta_i(k)$.

We may therefore restrict our attention to systems in which

$$\mathcal{L}(k) = \begin{bmatrix} 0 & \Delta_2(k) \\ \Delta_1(k) & 0 \end{bmatrix} \tag{9}$$

where $\Delta_i(k)$ has now the significance of probability to stay in the state i exactly for k time steps. By construction we have $\sum_{l=1}^{\infty}\Delta_i(l) = 1$ so that, from (4) we also get:

$$\mathcal{K}_{i'i''}(1) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

and thus, from (4) and defining $T_i = \sum_{l=1}^{\infty} l\Delta_i(l)$, we have:

$$\mathbf{j}(0) = \frac{(1,1)}{T_1 + T_2} \qquad \tilde{\mathbf{j}}(1) = \frac{(T_1, T_2)}{T_1 + T_2}$$

while simple calculations give

$$(\mathcal{I} - \tilde{\mathcal{L}}(z))^{-1} = \frac{1}{1 - \tilde{\Delta}_1(z)\tilde{\Delta}_2(z)}\begin{bmatrix} 1 & \tilde{\Delta}_2(z) \\ \tilde{\Delta}_1(z) & 1 \end{bmatrix}$$

To reflect the definition of Q we may now define $\mathbf{f} = (1, 0)$ and note that $C(k) = \mathbf{f}^t\mathcal{H}(k)\mathbf{f} - P_{ON}^2 = (\mathbf{f} - P_{ON})^t\mathcal{H}(k)(\mathbf{f} - P_{ON})$ where $P_{ON} = T_1/(T_1 + T_2)$ and thus

$$\begin{aligned} \tilde{C}(z) &= z(z-1)^{-2}\left(\tilde{\Delta}_1(z)\tilde{\Delta}_2(z) - 1\right)^{-1}(T_1 + T_2)^{-2} \\ &\quad \times \Big[(\tilde{\Delta}_1(z) - 1)(\tilde{\Delta}_2(z) - 1)(T_1 + T_2) \\ &\quad + (z-1)(\tilde{\Delta}_1(z)\tilde{\Delta}_2(z) - 1)T_1T_2\Big] \end{aligned} \tag{10}$$

To investigate the asymptotic behavior, let us review the following Tauberian result [16]:

Theorem 2: Let $\tilde{x}(z)$ be the z transform of the sequence x_k.

- If a continuous function $x\colon \mathbb{R}^+ \mapsto \mathbb{R}^+$ exists such that $x_k = x(k)$ and $x(t)$ is asymptotically equivalent to t^{2H-2} $(H \in]0.5, 1[)$ then $\tilde{x}(e^{\epsilon})$ converges for $\epsilon > 0$ while it diverges as ϵ^{1-2H} for $\epsilon \to 0^+$.
- If $\tilde{x}(e^{\epsilon}) \sim \epsilon^{1-2H}$ for $\epsilon \to 0^+$ and x_k is non-negative and eventually monotonic decreasing then $x_k \sim k^{2H-2}$ for $k \to \infty$.

Proof: Note first that for $\epsilon > 0$ we have $z^{-1} = e^{-\epsilon} < 1$ so that $\tilde{x}(e^{\epsilon}) = \sum_{k=0}^{\infty} x_k \left(z^{-1}\right)^k$ surely converges.

Define now the following subset of the real line $A(s) = \{\xi = k\epsilon | 0 \le \xi \le s\}$ and note that we can rewrite the z-transform as

$$\tilde{x}(e^{\epsilon}) = \frac{1}{\epsilon}\lim_{s\to\infty}\sum_{\xi\in A(s)} x_{\xi/\epsilon}e^{-\xi}\epsilon$$

which is asymptotically equivalent to the Riemann's sum of the corresponding integral with step ϵ. Hence the sum itself can be rewritten in the limit as

$$\tilde{x}(e^{\epsilon}) \sim \frac{1}{\epsilon}\int_0^{\infty} x\left(\frac{\xi}{\epsilon}\right)e^{-\xi}d\xi$$

We may now assume that $x(t)$ is negligibly different from the asymptotic behavior Xt^{2H-2} for $t > \bar{t}$. With this, the above asymptotic equivalence can be recast into

$$\begin{aligned} &\tilde{x}(e^{\epsilon}) \\ &\quad \sim \frac{1}{\epsilon}\int_0^{\bar{t}\epsilon} x\left(\frac{\xi}{\epsilon}\right)e^{-\xi}d\xi + \frac{X}{\epsilon}\int_{\bar{t}\epsilon}^{\infty}\left(\frac{\xi}{\epsilon}\right)^{2H-2}e^{-\xi}d\xi \\ &\quad = \int_0^{\bar{t}} x(t)e^{-t\epsilon}dt - X\epsilon^{1-2H}\left[\xi^{2H-1}\mathrm{E}_{2-2H}(\xi)\right]_{\bar{t}\epsilon}^{\infty} \end{aligned}$$

where the exponential integral function is defined as $\mathrm{E}_{2-2H}(\xi) = \int_1^{\infty} e^{-\xi a}a^{2H-2}da$. One may now check that the exponential integral function is such that $\xi^{2H-1}\mathrm{E}_{2-2H}(\xi) \to 0$ for $\xi \to \infty$ while $(\bar{t}\epsilon)^{2H-1}\mathrm{E}_{2-2H}(\bar{t}\epsilon) = \bar{t}^{2H-1}\Gamma(2H-1)$ for $\epsilon \to 0$, $\Gamma(\cdot)$ being the conventional gamma function.

Hence, taking the limit of the above expression for

$\epsilon \to 0$ one finally obtains

$$\tilde{x}(e^{\epsilon}) \sim \int_0^{\bar{t}} x(t)dt + X\epsilon^{1-2H}\bar{t}^{2H-1}\Gamma(2H-1) \sim \epsilon^{1-2H}$$

Conversely, we may exploit basic Tauberian theory (see e.g. [16, Theorem 8.7]) to obtain that, under our assumptions with the exception of the eventually decrease of x_k

$$\sum_{i=0}^{n} x_i \sim \frac{n^{2H-1}L(n)}{\Gamma(2H)}$$

where $L(n)$ is as slowly varying function for $n \to \infty$.

By adding the eventually decreasing property we know that $L(n)$ accounts for no oscillation around the asymptotic trend and are allowed to write

$$x_k \sim \sum_{i=0}^{k} x_i - \sum_{i=0}^{k-1} x_i \sim \frac{k^{2H-2}}{\Gamma(2H-1)}$$

□

Hence, we analyze the z-transform of Δ_i in the special case $z = e^{\epsilon}$ and $\epsilon \to 0$. In that neighborhood we may obtain an expansion of a generic $\tilde{\Delta}_i(z)$ noting that $\tilde{\Delta}(1) = 1$, that $\tilde{\Delta}'_i(1) = -T_i$ and that:

$$\tilde{\Delta}''_i(z) = z^{-2}\sum_{k=1}^{\infty} k(k-1)\Delta_i(k)z^{-k}$$

Hence, the behavior of $\tilde{\Delta}''_i(z)$ for $\epsilon \to 0$ depends on the asymptotic trend of $k^2\Delta_i(k)$.

If we assume a polynomially vanishing $\Delta_i(k) \sim k^{2H_i-4}$, with $H_i \in]0.5, 1[$, we have $k^2\Delta_i(k) \sim k^{2H_i-2}$ and thus, from Theorem 2, for $z = e^{\epsilon}$ and $\epsilon \to 0$ we have $\tilde{\Delta}''_i(z) \sim \epsilon^{1-2H_i}$ which accounts for an expansion of the kind:

$$\tilde{\Delta}_i(z) \sim 1 - T_i(z-1) + U_i(z-1)^{3-2H_i}$$

for some constant U_i. With the aim of comparison, let us observe that in the case of exponentially vanishing function, the same expression hold with $H_i = 0.5$.

Note that, if $H_i \in]0.5, 1[$ then $3-2H_i \in]1, 2[$ so that we may characterize the behavior of the z-transform or either exponentially or polynomially decaying $\Delta_i(k)$ with the three parameters T_i, U_i and $\alpha_i \in]1, 2]$ such that, when $z = e^{\epsilon}$ and $\epsilon \to 0$:

$$\tilde{\Delta}_i(z) \sim 1 - T_i(z-1) + U_i(z-1)^{\alpha_i} = 1 - T_i\epsilon + U_i\epsilon^{\alpha_i}$$

where we exploited also the asymptotic equivalence $e^{\epsilon} - 1 \sim \epsilon$. With these expansions (10) may be recast to:

$$\tilde{C}(z) \sim \frac{T_2^2 U_1 \epsilon^{\alpha_1 - 2} + T_1^2 U_2 \epsilon^{\alpha_2 - 2}}{(T_1 + T_2)^2}$$

With this, we may set $\alpha = \min_i\{\alpha_i\}$ to obtain:

$$\tilde{C}(z) \sim \epsilon^{\alpha-2}\frac{1}{(T_1+T_2)^2}\sum_{\alpha_i = \alpha} U_i T_{3-i}^2$$

Note finally that, if $\alpha < 2$, and the autocovariance is eventually positive and monotonic, Theorem 2 implies that it obeys:

$$C(k) \sim k^{1-\alpha} = k^{2H-2}$$

where H is related to the slowest asymptotic decay in sojourn time probabilities. Hence, when at least one state has a polynomially decaying sojourn time probability the Pseudo-Markov system is able to produce second-order self-similar trajectories. The scaling parameter of such trajectories is controlled by the slowest asymptotic decay in sojourn time probabilities.

Let us note that the autocovariance function has the same expression of (3).

5. Map Design Procedure

In this section we use the above self-similarity criteria to build a chaotic map generating self-similar trajectories.

In order to explain the map design procedure let us rename with p_k and q_k the probabilities of staying in the ON and in the OFF states for k steps, $\Delta_1(k)$ and $\Delta_2(k)$, respectively. The theory in Sect. 4 permits to build a chaotic self-similar generator by selecting at least one of the probabilities p_k and q_k polynomially vanishing.

In particular we select two cases: the first with $p_k \sim A\gamma^k$ and $q_k \sim Bk^{2H-4}$ with $A, B, \gamma > 0$ and H the Hurst parameter; the second with $p_k \sim Ak^{2H_1-4}$ and $q_k \sim Bk^{2H_2-4}$ with $A, B > 0$ and $H = \max(H_1, H_2)$ the Hurst parameter. These two cases permit to design two different kinds of self-similar maps with two ON/OFF sojourn time distributions: *light/heavy*-tailed and *heavy/heavy*-tailed.

To give a project criterion for these maps, by obtaining sojourn time distributions that allow a tuning of the process average but are as close as possible to the desired one (especially in the asymptotic trend), an ad-hoc, Lagrangian based technique is introduced.

Let us recall the ON and OFF average times $T_1 = \sum_{k=1}^{\infty} kp_k$ and $T_2 = \sum_{k=1}^{\infty} kq_k$, respectively. We want to assign $P_{ON} = T_1/(T_1+T_2)$ minimizing the deviation of p_k and q_k from the nominal decay $\tilde{p}_k$ and $\tilde{q}_k$. Hence we must solve the following optimization problem:

$$\min \sum_{k=1}^{\infty}\left(\frac{p_k}{\tilde{p}_k} - 1\right)^2 + \left(\frac{q_k}{\tilde{q}_k} - 1\right)^2$$

$$\text{s.t.}(P_{ON} - 1)\sum_{k=1}^{\infty} kp_k + P_{ON}\sum_{k=1}^{\infty} kq_k = 0$$

$$\text{s.t.} \sum_{k=1}^{\infty} p_k = 1\ ,\ \sum_{k=1}^{\infty} q_k = 1\ ,\ p_k \geq 0\ ,\ q_k \geq 0 \quad (11)$$

To solve such a problem we first note that, since when inequality constraints are active they set the corresponding probability to be zero, the functional form of the solution of (11) can be obtained by considering only the equality constraints. With this we obtain that the optimal probabilities $\overset{*}{p}_k$ and $\overset{*}{q}_k$ may have only two different functional forms, namely:

$$\overset{*}{p}_k = \begin{cases} \hat{p}_k = \tilde{p}_k + \dfrac{\lambda_1(P_{ON}-1)k}{2}\tilde{p}_k^2 + \dfrac{\lambda_2}{2}\tilde{p}_k^2 \\ 0 \quad \text{if } \hat{p}_k < 0 \end{cases} \tag{12}$$

$$\overset{*}{q}_k = \begin{cases} \hat{q}_k = \tilde{q}_k + \dfrac{\lambda_1 P_{ON} k}{2}\tilde{q}_k^2 + \dfrac{\lambda_3}{2}\tilde{q}_k^2 \\ 0 \quad \text{if } \hat{q}_k < 0 \end{cases} \tag{13}$$

Regrettably the value of the Lagrange multipliers λ_1, λ_2 and λ_3 depend on the indexes for which $\hat{p}_k < 0$ and $\hat{q}_k < 0$ and a suitable procedure must be devised to solve the problem.

To this aim note first that, given the asymptotic positivity of $\hat{p}_k$ and $\hat{q}_k$, only a finite number of vanishing entries exist in $\overset{*}{p}_k$ and $\overset{*}{q}_k$.

Moreover, any generic minimization problem in a sequence space with equality ad positivity constraints may benefit from the following Theorem:

Theorem 3: Let the sequences $\mathbf{v} = \{v_k\}_{k=1}^{\infty}$ and $\mathbf{a}_i = \{a_{ik}\}_{k=1}^{\infty}$ for $i = 1, \ldots, n$ be given along with the real numbers b_i for $i = 1, \ldots, n$. Assume that the solution $\overset{*}{\mathbf{v}} = \{\overset{*}{v}_k\}_{k=1}^{\infty}$ of the minimization problem:

$$\min \sum_{k=1}^{\infty} (v_k - 1)^2$$

$$\text{s.t.} \sum_{k=1}^{\infty} a_{ik} v_k - b_i = 0 \quad \forall i,\ v_k \geq 0$$

exist. Let also $\hat{\mathbf{v}} = \{\hat{v}_k\}_{k=1}^{\infty}$ be the solution of the relaxation, without the constraint $v_k \geq 0$, which surely exists. If $\hat{\mathbf{v}} \neq \overset{*}{\mathbf{v}}$ then when $\hat{v}_k < 0$ we have $\overset{*}{v}_k = 0$.

Proof: For any two sequences $\mathbf{v}'$ and $\mathbf{v}''$ define $\langle \mathbf{v}', \mathbf{v}'' \rangle = \sum_{k=1}^{\infty} v'_k v''_k$ so that $\sum_{k=1}^{\infty}(v_k - 1)^2 = \langle \mathbf{v} - 1, \mathbf{v} - 1 \rangle$. Indicate with π the linear subspace identified by the equality constraints $\langle \mathbf{a}_i, \mathbf{v} \rangle - b_i = 0$ for $i = 1, \ldots, n$. By its own definition $\hat{\mathbf{v}}$ is such that $\langle \hat{\mathbf{v}} - 1, \mathbf{v} - \hat{\mathbf{v}} \rangle = 0$ for every $\mathbf{v} \in \pi$. Hence, when $\mathbf{v} \in \pi$ we have $\langle \mathbf{v} - 1, \mathbf{v} - 1 \rangle = \langle \mathbf{v} - \hat{\mathbf{v}}, \mathbf{v} - \hat{\mathbf{v}} \rangle + \langle \hat{\mathbf{v}} - 1, \hat{\mathbf{v}} - 1 \rangle$.

Consider now a feasible point $\mathbf{v} \in \pi$ with $v_k \geq 0$ and the the gradient $\mathbf{w} = \nabla_{\mathbf{v}} \langle \mathbf{v} - \hat{\mathbf{v}}, \mathbf{v} - \hat{\mathbf{v}} \rangle = 2(\mathbf{v} - \hat{\mathbf{v}})$. Note that $\langle \mathbf{a}_i, \mathbf{w} \rangle = 0$ for $i = 1, \ldots, n$ so that we may obtain the feasible direction $\mathbf{w}'$ by computing $\mathbf{w}' = \mathbf{w} - \sum_{\substack{v_i = 0 \\ w_i > 0}} w_i \mathbf{e}_i$, where $\mathbf{e}_i$ is such that $e_{ii} = 1$ while $e_{ik} = 0$ for $i \neq k$.

The vector $\mathbf{w}'$ is such that if $v_k > 0$ and $\hat{v}_k < 0$ we have $w'_k > 0$ and that for some scalar $\phi > 0$ we have $\mathbf{v} - \phi\mathbf{w}' \in \pi$, $\mathbf{v} - \phi\mathbf{w}' \geq 0$ and $\langle \mathbf{v} - \hat{\mathbf{v}}, \mathbf{v} - \hat{\mathbf{v}} \rangle < \langle \mathbf{v} - \phi\mathbf{w}' - \hat{\mathbf{v}}, \mathbf{v} - \phi\mathbf{w}' - \hat{\mathbf{v}} \rangle$.

Hence, if $\hat{v}_k < 0$ the minimum of $\langle \mathbf{v} - \hat{\mathbf{v}}, \mathbf{v} - \hat{\mathbf{v}} \rangle$ (and thus of $\langle \mathbf{v} - 1, \mathbf{v} - 1 \rangle$) cannot be achieved at any point $\overset{*}{\mathbf{v}}$ such that $\overset{*}{v}_k > 0$. □

Note that we may rewrite (11) to fit the assumptions of Theorem 3 if we set $v_{2k-1} = p_k/\tilde{p}_k$ and $v_{2k} = q_k/\tilde{q}_k$ that leave the positivity constraints unchanged.

We may now address the solution of (11) and solve the relaxed problem where the Lagrange's multipliers $\lambda_1, \lambda_2, \lambda_3$ are determined by the satisfaction of the three constraints in (11).

This procedure must be iterated until the solution of the relaxed problem has no negative components.

Note that termination is guaranteed from the finiteness of the number of vanishing probabilities in the solution of (11) and from the fact that, when the solution of the relaxed problem has no negative components then it coincides with the solution of the non-relaxed problem.

Once that the two probabilities distributions $\overset{*}{p}_k$ and $\overset{*}{q}_k$ are known we may construct a chaotic map f whose iteration causes the state $x \in [0, 1]$ to switch between the ON condition $x \in [0, 1/2]$ and the OFF condition $x \in]1/2, 1]$ with the given statistics for the sojourn times.

5.1 Light/Heavy-Tailed ON/OFF Sojourn Time Map

To obtain a chaotic map with *light/heavy*-tailed sojourn profiles we set: $\tilde{p}_k \sim A\gamma^k$ and $\tilde{q}_k \sim Bk^{2H-4}$ according with the second criterion. Thus, by following the described map design criterion we obtain the map in Fig. 2.

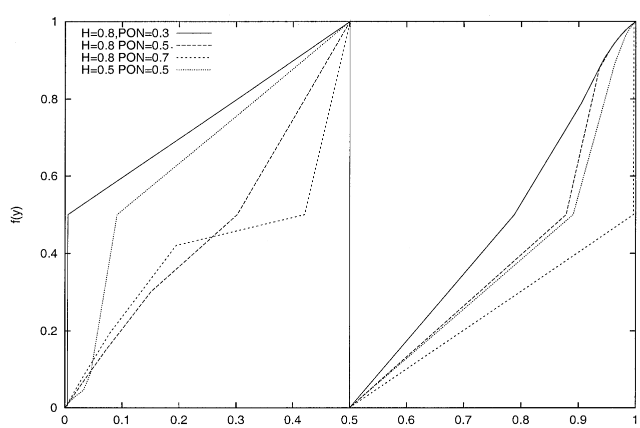

Fig. 2 Examples of the *light/heavy*-tailed chaotic map.

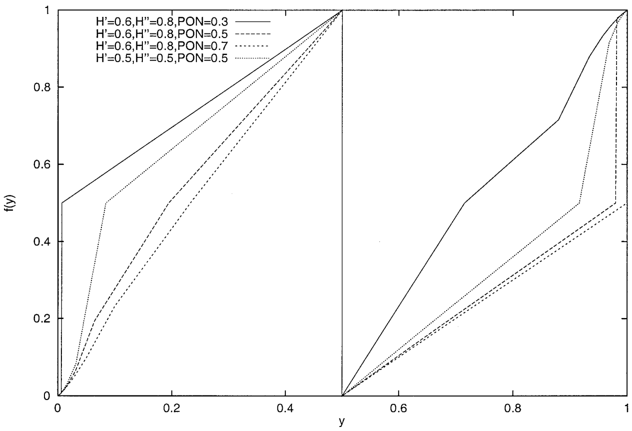

Fig. 3 Examples of the *heavy/heavy*-tailed chaotic map.

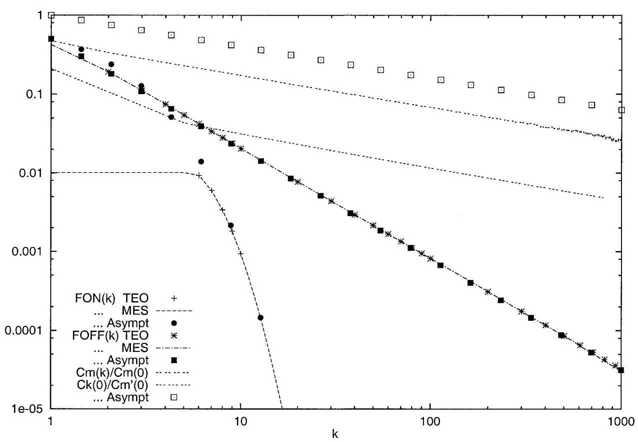

Fig. 4 ON/OFF sojourn time distributions (theoretical, measured and asymptotic), autocovariance functions (measured and asymptotic) relative to the *light/heavy*-tailed chaotic map, with $H = 0.8$ and $P_{ON} = 0.3$.

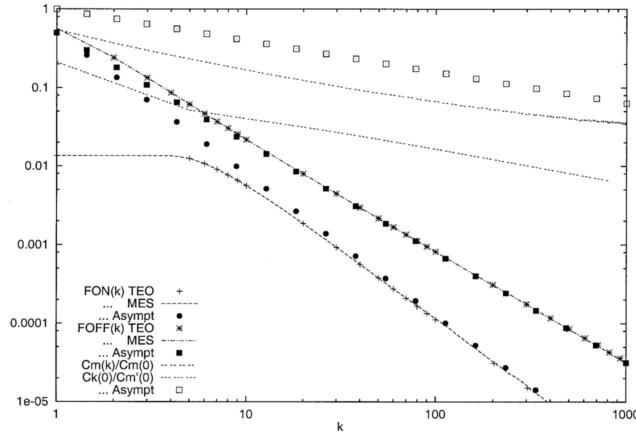

Fig. 5 ON/OFF sojourn time distributions (theoretical, measured and asymptotic), autocovariance functions (measured and asymptotic) relative to the *heavy/heavy*-tailed chaotic map, with $H' = 0.6, H'' = 0.8$ and $P_{ON} = 0.3$.

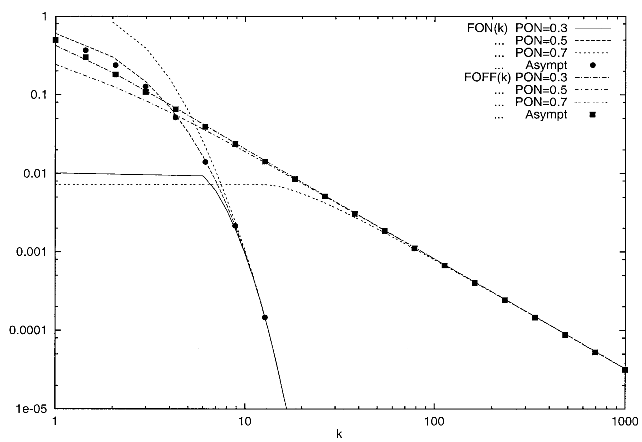

Fig. 6 ON/OFF sojourn time distributions (theoretical and asymptotic) relative to the *light/heavy*-tailed chaotic map, with $H = 0.8$ and $P_{ON} = 0.3, 0.5, 0.7$.

5.2 Heavy/Heavy-Tailed ON/OFF Sojourn Time Map

To obtain a chaotic map with *heavy/heavy*-tailed sojourn profiles we set: $\tilde{p}_k \sim Ak^{2H'-4}$ and $\tilde{q}_k \sim Bk^{2H''-4}$ according with the second criterion. Thus, by following the described map design criterion we obtain the map in Fig. 3.

6. Numerical Results

Some investigations have been performed to verify the behavior of the ON/OFF sojourn distributions of the proposed maps. In particular for both the cases *light/heavy*-tailed and *heavy/heavy*-tailed the theoretical, measured and asymptotic trends have been simulated, has reported in Figs. 4 and 5, where the complementary distributions of the ON time, $F_{ON}(k)$, of the OFF time, $F_{OFF}(k)$ and the autocovariance functions, $C^{(m)}(k)/C^{(m)}(0)$ and $C^{(k)}(0)/C^{(m')}(0)$ (m has been substituted by k to include this function in the same graph), are reported. The theoretical trend has been derived by means of the Lagrange-based iterative procedure described in Sect. 5; the measured trend has been obtained by iterating the generated maps and the asymptotic trend is the target trend for $k \gg 1$. The perfect match between the curves can be verified.

To numerically validate the proposed approach the autocovariance function has been also reported, by verifying that the maps produce a self-similar traffic. Let us observe that the asymptotic trends match perfectly with the simulations.

In Figs. 6 and 7 only the ON/OFF sojourn distributions have been reported (theoretical and asymptotic trend), by considering different P_{ON}.

In all cases the Hurst parameter has been set to $H = 0.8$ to simulate a high self-similar degree. Let us finally note that the described procedure allows to set independently P_{ON} and H, to simulate all possible

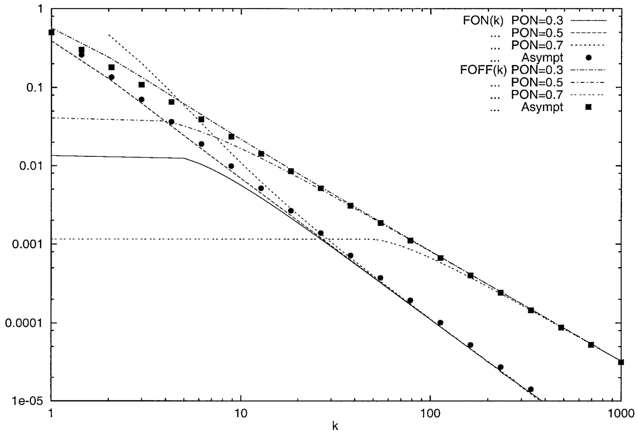

Fig. 7 ON/OFF sojourn time distributions (theoretical and asymptotic) relative to the *heavy/heavy*-tailed chaotic map, with $H' = 0.6, H'' = 0.8$ and $P_{ON} = 0.3, 0.5, 0.7$.

traffic conditions.

7. Conclusions

By starting from the definition of second-order self-similar processes, a methodology to design chaotic maps able to produce self-similar trajectories with any Hurst and activity index factors, has been presented. This methodology is based on three steps: we first show how the autocovariance of the process must be in order to match the second-order self-similar definition (Criterion I); then we have established a link between the autocovariance function and the probability of staying in the ON or in the OFF states (Criterion II); finally, we have explained a method to construct a map in which we can independently set the asymptotic trend of sojourn times and the average activity.

The numerical results show that the developed methodology allows to design a second-order self-similar generator with good performance. Further works are oriented to generalize the presented approach in order to consider environment with more than two states and higher order statistics.

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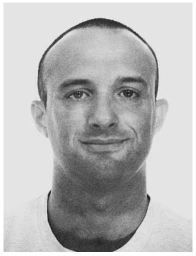

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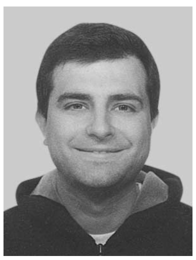

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